

COMPUTER-AIDED DESIGN OF STEPPED QUARTER
WAVE TRANSMISSION LINE FILTERS

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A Major Technical Report

in

The Faculty

of

Engineering

Presented in Partial Fulfillment of the Requirements
for the Master of Engineering Degree at
Concordia University
Montreal, Canada

September, 1979

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ABSTRACT

This report considers the design of a Stepped Quarter Wave Transformer with and without the capacitances formed at the junctions due to discontinuity.

With the help of scattering matrices, the resultant matrix is derived for n sections of transmission lines in cascade, each having the same length but different characteristic impedances. Power Loss Ratio is obtained from this expression and it is compared with the Butterworth and Chebyshev polynomials. It is shown that the resulting equations can be solved by the help of a computer.

Capacitances formed at the junctions due to discontinuity are then studied, its effects are analyzed and methods are applied to correct these effects. The Stepped Quarter Wave Transformer is then designed in this case using the computer program.

It has been found that any analytical solution is highly difficult and therefore computer aided solution has been resorted to for both the cases. This report includes samples of computer plots as well as the main program.

ACKNOWLEDGMENT .

The author wishes to express his deep sense of gratitude to Professor V. Ramachandran for suggesting the problem, his complete cooperation and understanding throughout the course of this investigation.

TABLE OF CONTENTS

	Page
ACKNOWLEDGMENT	iv
LIST OF TABLES	vii
LIST OF FIGURES	ix
Chapter	
1. INTRODUCTION	1
1.1 GENERAL	1
1.2 QUARTER WAVE TRANSFORMER	3
1.3 CAPACITANCES AT JUNCTIONS OF STEPPED QUARTER WAVE TRANSFORMERS	7
1.4 SCOPE OF THE REPORT	9
2. ANALYSIS OF STEPPED QUARTER WAVE TRANSFORMER	12
2.1 DISCONTINUOUS NON-UNIFORM LINES	12
2.2 SCATTERING MATRIX OF A UNIFORM TRANSMISSION LINE	20
2.3 CHARACTERISTICS OF MAXIMALLY FLAT TRANSFORMER	27
2.4 DESIGN OF STEPPED QUARTER WAVE TRANSFORMER FOR THE CASE $n = 3$	27
2.5 CHARACTERISTICS OF CHEBYSHEV TRANSFORMER	37
2.6 DESIGN OF STEPPED QUARTER WAVE TRANSFORMER FOR THE CASE $n = 3$	54
2.7 DISCUSSION	58
3. ANALYSIS OF STEPPED QUARTER WAVE TRANSFORMERS WITH CAPACITORS AT JUNCTIONS	64

	Page
3.1 EFFECTS OF DISCONTINUITY SUSCEPTANCES	64
3.2 METHODS OF CORRECTION FOR THESE EFFECTS	64
3.3 BUTTERWORTH CASE	69
3.4 CHEBYSHEV CASE	70
3.5 DISCUSSION	71
4. SUMMARY AND DISCUSSION	132
REFERENCES	134

LIST OF TABLES

Table	Page
2.1 The Reduction Formulas	29
2.2 Computer Program to Plot the Characteristics of Power Loss Ratio P_T versus θ for Butterworth Case	32
2.3 Computer Program to Find the Roots of the Equation (Butterworth Case)	36
2.4 Computer Program to Plot the Butterworth Characteristics Using the Values of Characteristic Impedances	38
2.5 Computer Program to Plot the Butterworth Characteristics Using Slightly Different Values of Characteristic Impedances	40
2.6 Computer Program to Calculate the Passband Tolerance ΔK	45
2.7 Computer Program to Plot the Characteristics of Power Loss Ratio P_T versus θ for Chebyshev Case	47
2.8 Computer Program to Find the Roots of the Equation (Chebyshev Case)	57
2.9 Computer Program to Plot the Chebyshev Characteristics Using the Values of Characteristic Impedances	59
2.10 Computer Program to Plot the Chebyshev Characteristics Using Slightly Different Values of Characteristic Impedances	61
3.1 Computer Program to Observe the Effect of Shunt Capacitance ($C = 10.0 \times 10^{-12}$) and to Calculate New Values of Transmission and Reflection Coefficients, Characteristic Impedances etc. (Butterworth Case)	72
3.2 Computer Program to Observe the Effect of Shunt Capacitance ($C = 15.0 \times 10^{-12}$) and to Calculate New Values of Transmission and Reflection Coefficients, Characteristic Impedances etc. (Butterworth Case)	82

Table.

Page

- 3.3 Computer Program to Observe the Effect of Shunt
Capacitance ($c = 20.0 \times 10^{-12}$) and to Calculate
New Values of Transmission and Reflection Coefficients,
Characteristic Impedances etc. (Butterworth Case) 92
- 3.4 Computer Program to Observe the Effect of Shunt
Capacitance ($c = 10.0 \times 10^{-12}$) and to Calculate New
Values of Transmission and Reflection Coefficients,
Characteristic Impedances etc. (Chebyshev Case)102
- 3.5 Computer Program to Observe the Effect of Shunt
Capacitance ($c = 15.0 \times 10^{-12}$) and to Calculate New
Values of Transmission and Reflection Coefficients,
Characteristic Impedances etc. (Chebyshev Case)112
- 3.6 Computer Program to Observe the Effect of Shunt
Capacitance ($c = 20.0 \times 10^{-12}$) and to Calculate New
Values of Transmission and Reflection Coefficients,
Characteristic Impedances etc. (Chebyshev Case)122

LIST OF FIGURES

Figure	Page
1.1 Quarter Wave Transformer	4
1.2 Stepped Quarter Wave Transformer	6
1.3 Capacitances at Junctions of Stepped Quarter Wave Transformers and Equivalent Circuit of a Single Step ...	8
2.1 Discontinuous Non-Uniform Line Transformer where the Characteristic Impedance Changes in Steps	13
2.2 Transmission and Reflection Coefficients from a Junction Discontinuity	14
2.3 The Outward Travelling Wave in the i^{th} Line when Other Lines Are matched	16
2.4 A Section of a Lossless Uniform Transmission Line	21
2.5 Transmission and Reflection Coefficients at Junction Discontinuity	23
c2.1 Response of Power Loss Ratio P_T against θ . Butterworth Case of Order 3	33
c2.2 Response of Power Loss Ratio P_T against θ Using the values of the Characteristic Impedances. Butterworth Case of Order 3	39
c2.3 Response of Power Loss Ratio P_T against θ Using Slightly Changed Values of the Characteristic Impedances. Butterworth Case of Order 3	41
c2.4 Response of Power Loss Ratio P_T against θ . Chebyshev Case of Order 3, Scale Factor $p = 0.86$, Passband Tolerance $AK = 0.10201295$	48
c2.5 Response of Power Loss Ratio P_T against θ . Chebyshev Case of Order 3, Scale Factor $p = 0.87$, Passband Tolerance $AK = 0.11600182$	49

Figure	Page
c2.6 Response of Power Loss Ratio P_T against θ . Chebyshev Case of Order 3, Scale Factor $p = 0.88$, Passband Tolerance $AK = 0.13213671$	50
c2.7 Response of Power Loss Ratio P_T against θ . Chebyshev Case of Order 3, Scale Factor $p = 0.89$, Passband Tolerance $AK = 0.15080594$	51
c2.8 Response of Power Loss Ratio P_T against θ . Chebyshev Case of Order 3, Scale Factor $p = 0.90$, Passband Tolerance $AK = 0.17248278$	52
c2.9 Response of Power Loss Ratio P_T against θ . Chebyshev Case of Order 3, Scale Factor $p = 0.91$, Passband Tolerance $AK = 0.19774788$	53
c2.10 Response of Power Loss Ratio P_T against θ Using the Values of the Characteristic Impedances. Chebyshev Case of Order 3	60
c2.11 Response of Power Loss Ratio P_T against θ Using Slightly Changed Values of the Characteristic Impedances. Chebyshev Case of Order 3	62
3.1 Stepped Quarter Wave Transformer with Capacitances at Junctions	65
3.2 Equivalent Circuit of a Single Step	67
c3.1 Response of Power Loss Ratio P_T against θ with Capacitance at Junction. Butterworth Case of Order 3, $c = 10.0 \times 10^{-12}$, $W = 1 \times 10^8$	75
c3.2 Response of Power Loss Ratio P_T against θ with Capacitance at Junction. Butterworth Case of Order 3, $c = 10.0 \times 10^{-12}$, $W = 2 \times 10^8$	77
c3.3 Response of Power Loss Ratio P_T against θ with Capacitance at Junction. Butterworth Case of Order 3, $c = 10.0 \times 10^{-12}$, $W = 4 \times 10^8$	79
c3.4 Response of Power Loss Ratio P_T against θ with Capacitance at Junction. Butterworth Case of Order 3, $c = 10.0 \times 10^{-12}$, $W = 5 \times 10^8$	81
c3.5 Response of Power Loss Ratio P_T against θ with Capacitance at Junction. Butterworth Case of Order 3, $c = 15.0 \times 10^{-12}$, $W = 1 \times 10^8$	85

Figure

Page

c3.6	Response of Power Loss Ratio P_T against θ with Capacitance at Junction. Butterworth Case of Order 3, $c = 15.0 \times 10^{-12}$, $W = 2 \times 10^8$	87
c3.7	Response of Power Loss Ratio P_T against θ with Capacitance at Junction. Butterworth Case of Order 3, $c = 15.0 \times 10^{-12}$, $W = 4 \times 10^8$	89
c3.8	Response of Power Loss Ratio P_T against θ with Capacitance at Junction. Butterworth Case of Order 3, $c = 15.0 \times 10^{-12}$, $W = 5 \times 10^8$	91
c3.9	Response of Power Loss Ratio P_T against θ with Capacitance at Junction. Butterworth Case of Order 3, $c = 20.0 \times 10^{-12}$, $W = 1 \times 10^8$	95
c3.10	Response of Power Loss Ratio P_T against θ with Capacitance at Junction. Butterworth Case of Order 3, $c = 20.0 \times 10^{-12}$, $W = 2 \times 10^8$	97
c3.11	Response of Power Loss Ratio P_T against θ with Capacitance at Junction. Butterworth Case of Order 3, $c = 20.0 \times 10^{-12}$, $W = 4 \times 10^8$	99
c3.12	Response of Power Loss Ratio P_T against θ with Capacitance at Junction. Butterworth Case of Order 3, $c = 20.0 \times 10^{-12}$, $W = 5 \times 10^8$	101
c3.13	Response of Power Loss Ratio P_T against θ with Capacitance at Junction. Chebyshev Case of Order 3, $c = 10.0 \times 10^{-12}$, $W = 1 \times 10^8$	105
c3.14	Response of Power Loss Ratio P_T against θ with Capacitance at Junction. Chebyshev Case of Order 3, $c = 10.0 \times 10^{-12}$, $W = 2 \times 10^8$	107
c3.15	Response of Power Loss Ratio P_T against θ with Capacitance at Junction. Chebyshev Case of Order 3, $c = 10.0 \times 10^{-12}$, $W = 4 \times 10^8$	109
c3.16	Response of Power Loss Ratio P_T against θ with Capacitance at Junction. Chebyshev Case of Order 3, $c = 10.0 \times 10^{-12}$, $W = 5 \times 10^8$	111
c3.17	Response of Power Loss Ratio P_T against θ with Capacitance at Junction. Chebyshev Case of Order 3, $c = 15.0 \times 10^{-12}$, $W = 1 \times 10^8$	115

c3.18	Response of Power Loss Ratio P_T against θ with Capacitance at Junction. Chebyshev Case of Order 3, $c = 15.0 \times 10^{-12}$, $W = 2 \times 10^8$	117
c3.19	Response of Power Loss Ratio P_T against θ with Capacitance at Junction. Chebyshev Case of Order 3, $c = 15.0 \times 10^{-12}$, $W = 4 \times 10^8$	119
c3.20	Response of Power Loss Ratio P_T against θ with Capacitance at Junction. Chebyshev Case of Order 3, $c = 15.0 \times 10^{-12}$, $W = 5 \times 10^8$	121
c3.21	Response of Power Loss Ratio P_T against θ with Capacitance at Junction. Chebyshev Case of Order 3, $c = 20.0 \times 10^{-12}$, $W = 1 \times 10^9$	125
c3.22	Response of Power Loss Ratio P_T against θ with Capacitance at Junction. Chebyshev Case of Order 3, $c = 20.0 \times 10^{-12}$, $W = 2 \times 10^9$	127
c3.23	Response of Power Loss Ratio P_T against θ with Capacitance at Junction. Chebyshev Case of Order 3, $c = 20.0 \times 10^{-12}$, $W = 4 \times 10^9$	129
c3.24	Response of Power Loss Ratio P_T against θ with Capacitance at Junction. Chebyshev Case of Order 3, $c = 20.0 \times 10^{-12}$, $W = 5 \times 10^9$	131

CHAPTER 1

INTRODUCTION

1.1 GENERAL

A transmission line is usually used for delivering power from a generator to a receiver which may be an impedance, a terminal impedance Z_R . We generally wish to design the line so as to transmit the maximum possible power from generator to receiver. It means there must be some conditions which must be satisfied by the line so as to transmit the maximum power.

It is clear that if the line has attenuation, there will be power lost along the line. Therefore the first condition for maximum power transfer is to have a line without attenuation. Theoretically it is possible but practically not since resistance cannot be avoided, but in a well designed microwave transmission line the attenuation is small and it can be assumed that there is no attenuation as an approximation. Now it is not necessary that a line without attenuation will transmit the maximum possible power from the generator to the receiver. For maximum power transfer, the impedance of the generator must be matched to that of the load. If the line has no losses, it means that the only power dissipation must be in the generator and the receiver, the complete power output of the generator, except for what is absorbed in its own internal resistance, is delivered to the receiver. The flow of power through any cross section of the line must then be the same

and equal to the flow of power into the receiver. Now if impedances are matched at one point of the line, it means the maximum possible power crosses this point and is delivered to the load. Further matching at other points must lead to the same condition and hence must be secured by matching at one point..

This does not mean, however, that the load itself must have an impedance that has any fixed relation to the impedance of the generator. The lossless transmission line connecting them has the properties of a transformer. The line can be replaced by a four terminal network. The power is then fed in from the generator at one voltage and current or with one impedance and it is fed out from the other end to the load with another voltage and current, or another impedance, since the line is lossless, no power is lost in the line therefore the product of the input voltage and current equals the product of the output voltage and current which is the characteristic of a "lossless transformer". Therefore a section of transmission line to connect a generator and load can be used as a "matching transformer" in order to satisfy the conditions of maximum power transfer. The purpose of the matching section is to transform the load impedance and present to the main line a terminating impedance equal to its characteristic impedance. As a consequence there are no standing waves on the main line and the transmission line losses are thus minimized. This technique of inserting a matching section to transform the load impedance to the characteristic impedance of the main transmission line

is known as transmission-line impedance matching.

1.2 QUARTER WAVE TRANSFORMER

The simple impedance matching device consists only of a series section of a transmission line known as a "Quarter-Wave Transformer", which is a quarter wave length of lossless line with characteristic impedance equal to the geometric mean of two resistances, and so designed that it matches these resistances to each other, so that a generator have a real impedance equal to the first resistance will be properly matched by the transformer to a load have a real impedance equal to a second resistance.

Let A represent the terminals of a long transmission line whose characteristic resistance is R_0 and let AB represent a section of a series transmission line, one pair of whose terminal is connected to the load resistor R_L and other pair to the terminals of the main line as shown in Fig. 1.1. R_{om} is the characteristic resistance and l is the length of the matching section of the transmission line. As the name of this impedance matching device implies, l is chosen to be equal to $\lambda/4$ at the frequency for which impedance match is required. Now in order for the equivalent load resistance terminating the main line to be equal to R_0 , the characteristic resistance R_{om} of the matching section should be

$$R_{om} = \sqrt{R_L R_0}$$

Thus, if between the main line and the load a quarter

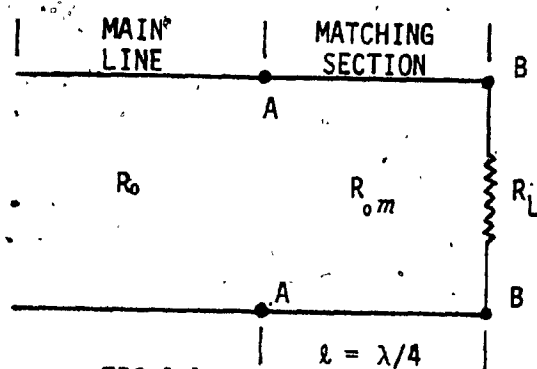


FIG 1.1

QUARTER WAVE TRANSFORMER

wavelength of a transmission line is inserted having a characteristic resistance equal to the geometric mean of the load resistance and the characteristic resistance of main line, the terminating resistance of the main line is equal to its characteristic resistance with the result that there are no standing waves on the main line.

One thing should be kept in mind here that the impedance match is achieved at only a single frequency for which the length of the matching section is a quarter wave length. At other frequencies, the length of the matching section is different from a quarter wavelength. It means that the impedance match obtained with the use of quarter wave transformer is frequency sensitive i.e., having matched at one frequency, a mismatch occurs at other frequencies.

Therefore what should be done is to achieve perfect matching over a range of frequencies. Combinations of the kind circuit elements can result in good broadband matching. One way of obtaining broadband operation is by the use of several transformer sections that introduce a change in impedance gradually. Thus several quarter wave sections of slightly different impedances may be employed which will make the overall device less sensitive to frequency because more gradual the change, the less the reflections and the less selective is the transformer.

Now when the characteristic impedance changes in steps, the transformer is called Stepped Quarter Wave Transformer as shown in Fig. 1.2. This quarter wave transformer consist of three lossless

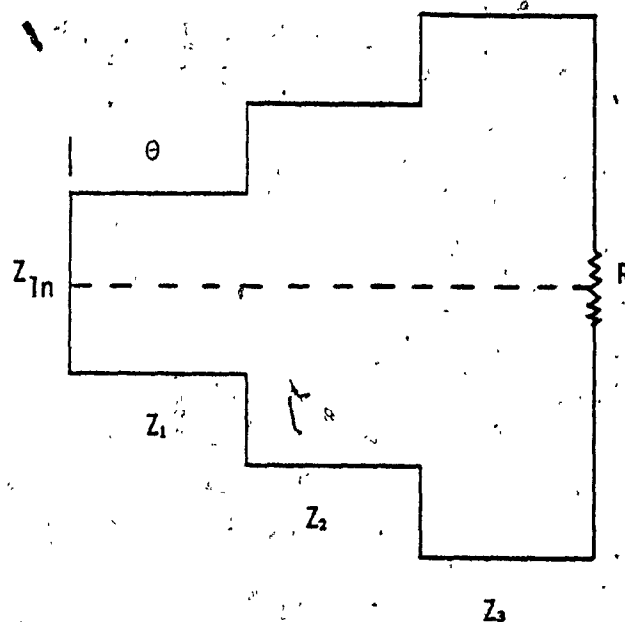


FIG 1.2

STEPPED QUARTER-WAVE TRANSFORMER

sections each of electrical length θ and physical length which is equal to $\lambda/4$ and terminated in a pure resistive load R . The characteristic impedances are Z_1 , Z_2 and Z_3 . These characteristic impedances are determined for the Stepped Quarter Wave Transformer to give Chebyshev and Butterworth behaviour in the passband.

Quarter wave transformers have numerous applications besides being impedance transformers, an understanding of their behaviour gives insight into many other physical situations not connected with impedance transformations. The design equations have been developed where they can be used for the synthesis of circuits such as reactance-coupled filters, short line low pass filters, branch guide directional couplers, optical multi-layer filters and transformers and acoustical transformers.

1.3 Capacitances at junctions of Stepped Quarter Wave Transformers

A discontinuity in a coaxial line cross-section cannot be represented by a change of impedance only i.e., practical junctions are non-ideal. Ideal junction is the connection between two impedances or transmission lines, when the electrical effects of the connecting wires, or the junction discontinuities, can be neglected.

It was assumed before that the discontinuity susceptances B_m in the stepped transformer are zero. This would be approximately true in a low frequency coaxial line, but not in a high frequency coaxial line. The presence of the discontinuity susceptances has two effects.

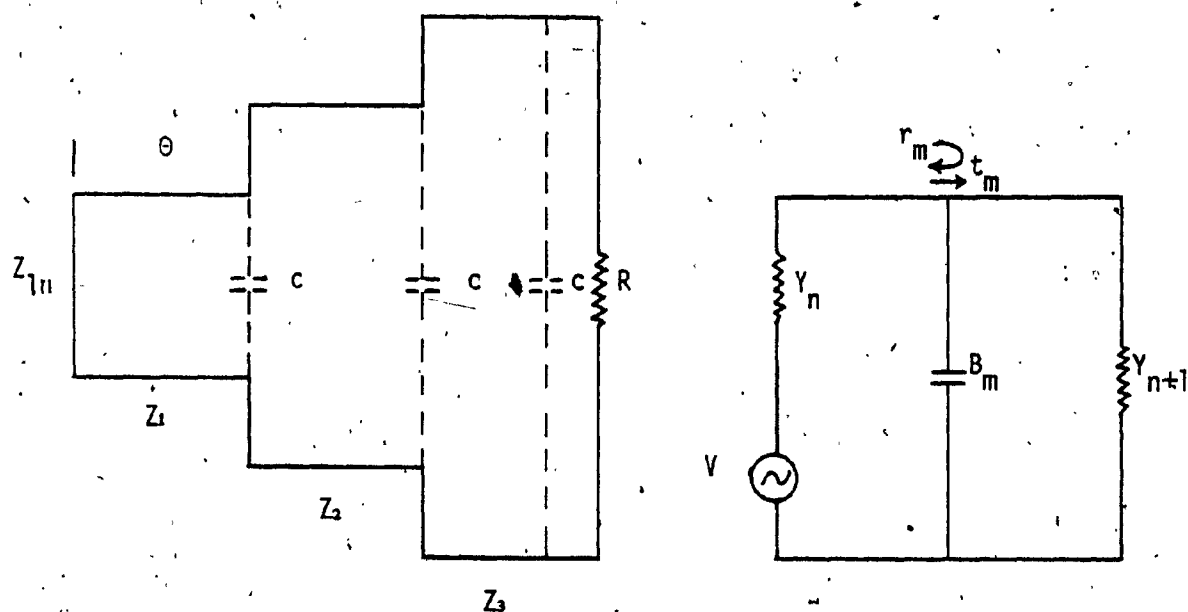


FIG 1.3

CAPACITANCES AT JUNCTIONS OF STEPPED QUARTER WAVE
TRANSFORMERS & EQUIVALENT CIRCUIT OF A SINGLE STEP

The lesser effect is the small increase in the magnitudes of the individual step reflections. The greater effect is the introduction of phase angles in the reflection and transmission coefficients of the steps.

These capacitive effects are analyzed and methods are applied to correct these effects. Then the characteristic impedances are determined to give Chebyshev and Butterworth behaviour in the passband.

1.4 SCOPE OF THE REPORT

This report examines the Stepped Quarter Wave Transformer and determines the characteristic impedances for three sections so that the load is perfectly matched and the maximum mismatch between the zeros in the passband is constant. The characteristic impedances are so obtained to give Butterworth and Chebyshev behaviour in the passband. The characteristic impedances are then determined for both Butterworth and Chebyshev cases taking into consideration the effect of discontinuity capacitances.

Chapter wise the report is as follows:

In Chapter II, the overall matrix for n sections of transmission lines in cascade, each having the same length but different characteristic impedance is obtained and the Power Loss Ratio is defined involving this matrix. Since in this report three sections are considered so a Power Loss Ratio for three sections is obtained. Now a Butterworth

polynomial of third degree is taken and characteristic impedances are calculated with the aid of a computer program using root finding subroutine. In this case passband tolerance is fixed so the characteristics of Power Loss Ratio are plotted using the plotting subroutine. After this a Chebyshev Polynomial of third degree is taken and characteristic impedances are calculated with the aid of computer subroutine for root finding. The passband tolerance AK is calculated by a computer program for different values of scale factor. The characteristics of Power Loss Ratio for different values of scale factor and Passband Tolerance are plotted with the aid of a computer plotting subroutine.

In Chapter III, the Stepped Quarter Wave Transformer with junction capacitances is considered and the effect of these capacitances is analyzed and a computer program is written which shows the effects at different frequencies for different capacitance values. With the help of this computer program the values of susceptance, extra phase shift introduced due to this capacitance and the distance moved in order to compensate for this capacitance are obtained. In addition with the help of this computer program the new values of Transmission and Reflection coefficients, the new values of characteristic impedances are obtained in this case to give Butterworth and Chebyshev behaviour in the passband. The plot of characteristics for both Butterworth and Chebyshev are also obtained with the help of this computer program.

Chapter IV discusses the conclusions arising out of this study.

CHAPTER II

ANALYSIS OF STEPPED QUARTER WAVE TRANSFORMER

The Stepped Quarter Wave Transformer is shown in Fig. 2.1. It consists of three sections each of electrical length θ and physical length which is equal to $\lambda/4$ and terminated in a pure resistive load R . The characteristic impedances are Z_1 , Z_2 and Z_3 which are determined for the Stepped Quarter Wave Transformer to give Butterworth and Chebyshev behaviour in the passband.

2.1 Discontinuous Non-Uniform Lines

Since between two unequal impedances Z_1 and Z_2 there is a junction discontinuity, so considering the general discontinuity problem.

Since reflection and transmission coefficients will be expressed in scattering matrix representation so first looking at the Scattering matrix of Microwave Junctions.

A microwave system consists of several components which are connected in some sequence to obtain the desired transmission or receiving characteristics. The interconnection of two or more microwave components or transmission lines may be regarded as a microwave junction.

The relation between the reflected and the incident wave is given by

$$V_1^r = \Gamma_1 V_1^i$$

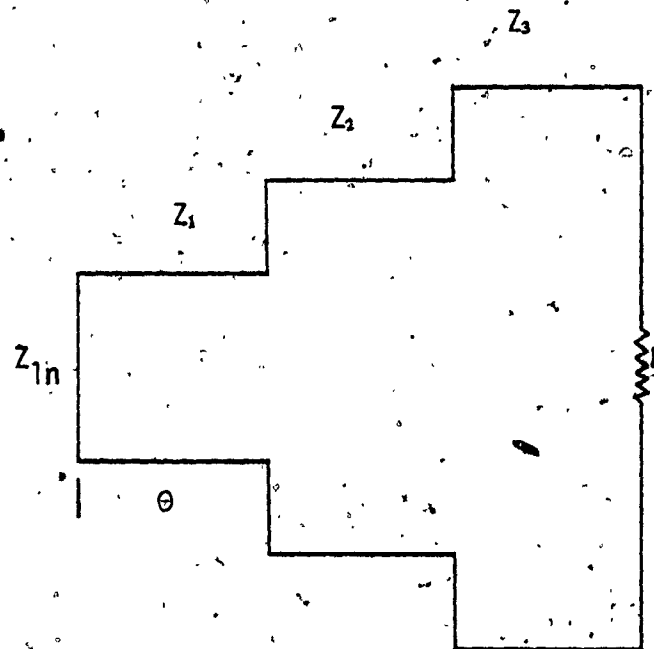


FIG 2.1

DISCONTINUOUS NON-UNIFORM LINE TRANSFORMER WHERE
THE CHARACTERISTIC IMPEDANCE CHANGES IN STEPS

C

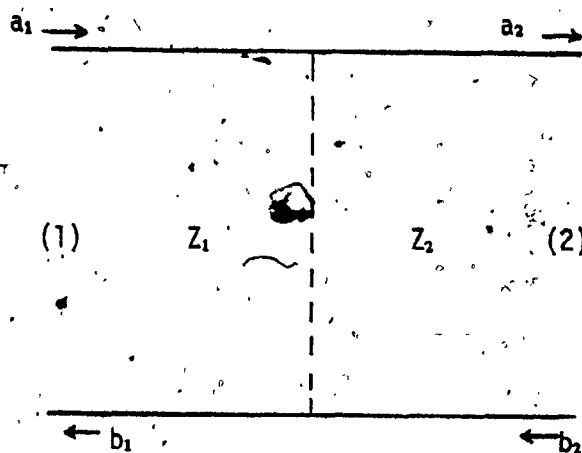


FIG 2.2

TRANSMISSION AND REFLECTION COEFFICIENTS FROM A
JUNCTION DISCONTINUITY

Where subscripts r and i refer to the reflected and incident waves, respectively. Γ_i is the reflection coefficient of the i^{th} line.

If a generator is connected only at the i^{th} line and every other line at the junction is terminated by a matched impedance as shown in Fig. 2.3, the outward travelling wave (coming outward from the junction) represented by b_i can be expressed as

$$b_i = \Gamma_i a_i$$

where a_i is the incident travelling wave coming towards the junction.

If there are some incident waves (a_1, a_2 , etc.) coming from lines 1, 2, 3,, the outward travelling wave of the i^{th} line becomes

$$b_i = S_{i1} a_1 + S_{i2} a_2 + S_{i3} a_3 + \dots S_{in} a_n \quad \dots (2.1)$$

In equation (2.1) $S_{ij} a_j$ represents the contribution of the outward travelling wave at the i^{th} line due to the incident wave a_j at the line j . Similarly, $S_{i2} a_2, S_{i3} a_3$, etc. represent the contributions of the incident waves a_2, a_3 , etc. in lines 2, 3,, to the outward travelling wave at the line i .

S_{ii} represents the reflection coefficient of the i^{th} line

and $S_{ii} = \Gamma_i$

The justification of algebraic summation of all the contributions from different lines is directly dependent upon the theory of linear superposition. So equation (2.1) can be written as

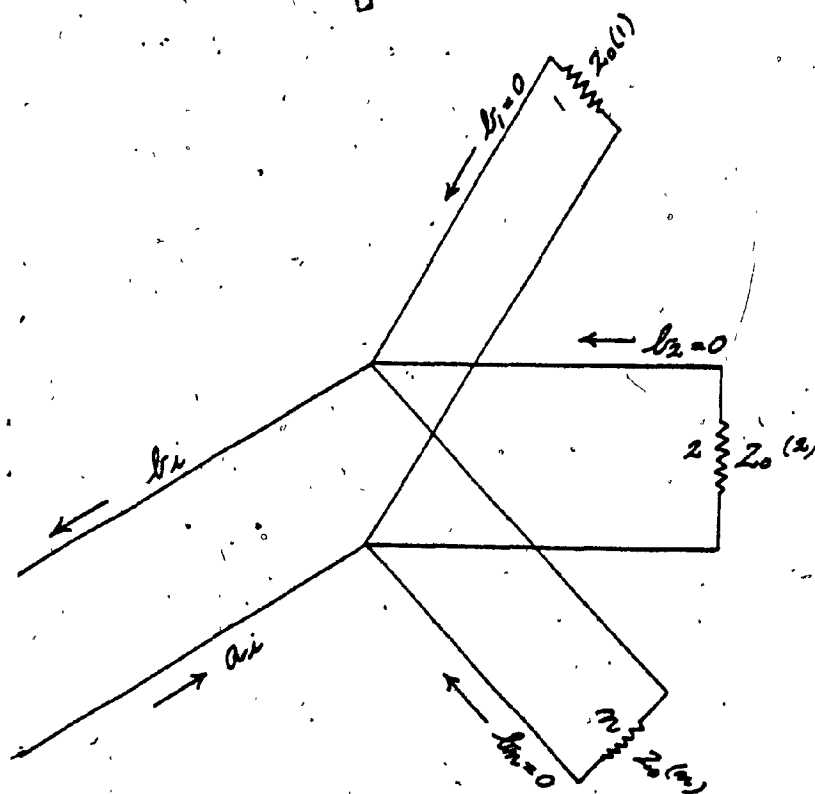


FIG 2.3

THE OUTWARD TRAVELLING WAVE IN THE 1th LINE WHEN
OTHER LINES ARE MATCHED

$$b_1 = S_{11} a_1 + S_{12} a_2 + S_{13} a_3 + \dots S_{1n} a_n$$

$$b_2 = S_{21} a_1 + S_{22} a_2 + S_{23} a_3 + \dots S_{2n} a_n$$

$$\dots\dots\dots$$

$$b_n = S_{n1} a_1 + S_{n2} a_2 + S_{n3} a_3 + \dots S_{nn} a_n$$

In matrix notation

$$\hat{\mathbf{b}} = \hat{\mathbf{S}} \hat{\mathbf{a}} \quad \dots(2.2)$$

where both \mathbf{b} and \mathbf{a} are column matrices. The n and n matrix \mathbf{S} is called the scattering matrix, and the coefficients S_{11}, S_{12}, \dots etc are called the scattering coefficients. Scattering means reflection in some form or other

Recalling figure (2.2) and seeing equation (2.2) we have

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

But here $b_2 = a_2$

therefore

$$\begin{bmatrix} b_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} \Gamma_1 & T_2 \\ T_1 & \Gamma_2 \end{bmatrix} \begin{bmatrix} a_1 \\ b_2 \end{bmatrix}$$

That is if T_1 and T_2 are respectively, the transmission coefficients of the discontinuity for the waves incident from the lines 1 and 2, and if Γ_1 and Γ_2 are the corresponding reflection coefficients of the discontinuity, the scattering matrix parameter of the junction may be written as shown above. It can also be written as

$$b_1 = \Gamma_1 a_1 + T_2 b_2$$

$$a_2 = \Gamma_2 b_1 + T_1 a_1$$

where

$$\Gamma_1 = \frac{Z_2 - Z_1}{Z_1 + Z_2}$$

and

$$\Gamma_2 = -\Gamma_1 = \frac{Z_1 - Z_2}{Z_1 + Z_2}$$

and

$$T_1 = 1 + \Gamma_1 = 1 + \frac{Z_2 - Z_1}{Z_1 + Z_2}$$

$$T_1 = \frac{2Z_2}{Z_1 + Z_2}$$

$$T_2 = 1 + \Gamma_2 = 1 + \frac{Z_1 - Z_2}{Z_1 + Z_2} = \frac{2Z_1}{Z_1 + Z_2}$$

$$T_2 \neq T_1$$

For the junction of the transmission line, therefore we can write the equations in the form of a_1 and b_1 in terms of a_2 and b_2 .

Now we have

$$b_1 = \Gamma_1 a_1 + T_2 b_2 \quad \dots (2.3)$$

and

$$a_2 = \Gamma_2 b_2 + T_1 a_1 \quad \dots (2.4)$$

From (2.4)

$$a_1 = a_2 / T_1 - \Gamma_2 / T_1 b_2$$

or

$$a_1 = \frac{Z_1 + Z_2}{2Z_2} a_2 + \frac{Z_2 - Z_1}{2Z_2} b_2$$

From (2.3)

$$b_1 = \frac{Z_2 - Z_1}{Z_1 + Z_2} \left\{ \frac{Z_1 + Z_2}{2Z_2} a_2 + \frac{Z_2 - Z_1}{2Z_2} b_2 \right\} + \frac{2Z_1}{Z_1 + Z_2} b_2$$

$$b_1 = \frac{(Z_2 - Z_1) a_2}{2Z_2} + \frac{(Z_1 + Z_2) b_2}{2Z_2}$$

$$a_1 = \frac{Z_1 + Z_2}{2Z_2} a_2 + \frac{Z_2 - Z_1}{2Z_2} b_2$$

and

$$b_1 = \frac{Z_2 - Z_1}{2Z_2} a_2 + \frac{(Z_1 + Z_2) b_2}{2Z_2}$$

therefore

$$\begin{bmatrix} a_1 \\ b_1 \end{bmatrix} = \frac{1}{2Z_2} \begin{bmatrix} Z_1 + Z_2 & Z_2 - Z_1 \\ Z_2 - Z_1 & Z_1 + Z_2 \end{bmatrix} \begin{bmatrix} a_2 \\ b_2 \end{bmatrix} \quad \dots (2.5)$$

2.2 Scattering Matrix of a Uniform Transmission Line

Considering the scattering matrix of a section of uniform lossless transmission line of length L , as shown in Fig. (2.4) this section can be regarded as a two port junction with incident voltages a_1, a_2 and reflected voltages b_1, b_2 for ports 1 and 2, respectively. The terminal planes 1 and 2 of the junction are assumed to be as shown in Fig. (2.4). Since

$$b_2 = a_1 e^{-j\beta L} \quad \text{and} \quad b_1 = a_2 e^{-j\beta L}$$

In terms of a_2 and b_2 equations can be written as

$$\text{and} \quad \left. \begin{aligned} b_2 &= a_1 e^{-j\theta} \\ a_2 &= b_1 e^{j\theta} \end{aligned} \right\} \text{ as } \beta L = \theta$$

In matrix form we can write

$$\begin{bmatrix} b_2 \\ a_2 \end{bmatrix} = \underbrace{\begin{bmatrix} e^{-j\theta} & 0 \\ 0 & e^{j\theta} \end{bmatrix}}_{\text{TRANSMISSION MATRIX (T}_L\text{)}} \begin{bmatrix} a_1 \\ b_1 \end{bmatrix}$$

$$\begin{bmatrix} a_1 \\ b_1 \end{bmatrix} = \underbrace{\begin{bmatrix} e^{j\theta} & 0 \\ 0 & e^{-j\theta} \end{bmatrix}}_{T_L^{-1}} \begin{bmatrix} b_2 \\ a_2 \end{bmatrix}$$

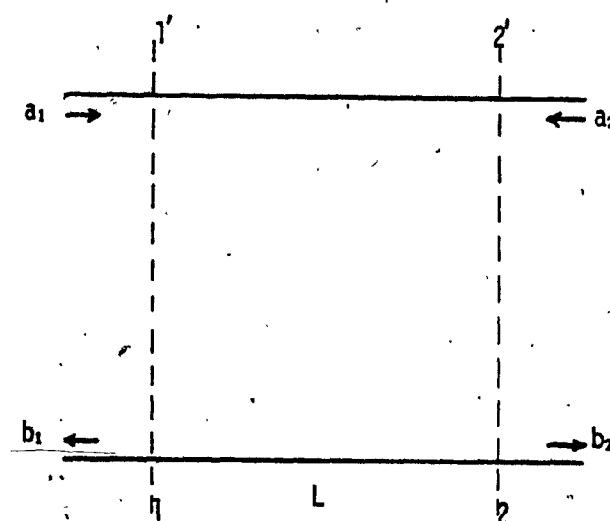


FIG 2.4

A SECTION OF A LOSSLESS UNIFORM TRANSMISSION LINE

Therefore the inverse transmission matrix of a uniform line of length θ is

$$T_L^{-1} = \begin{bmatrix} e^{j\theta} & 0 \\ 0 & e^{-j\theta} \end{bmatrix}$$

Now at the port 2 of Fig. (2.4) we have incident voltage a_2 and reflected voltage b_2 , so this will form the input at the junction discontinuity as shown in Fig. 2.5.

Therefore $b_2 = a_1$ and $a_2 = b_1$, so putting these values in expression (2.5)

$$\begin{bmatrix} b_2 \\ a_2 \end{bmatrix} = \frac{1}{2Z_2} \begin{bmatrix} Z_1 + Z_2 & Z_2 - Z_1 \\ Z_2 - Z_1 & Z_1 + Z_2 \end{bmatrix} \begin{bmatrix} b_1 \\ a_1 \end{bmatrix}$$

or

$$\begin{bmatrix} a_2 \\ b_2 \end{bmatrix} = \frac{1}{2Z_2} \underbrace{\begin{bmatrix} Z_1 + Z_2 & Z_2 - Z_1 \\ Z_2 - Z_1 & Z_1 + Z_2 \end{bmatrix}}_{T_D} \begin{bmatrix} a_1 \\ b_1 \end{bmatrix}$$

therefore

$$\begin{bmatrix} a_1 \\ b_1 \end{bmatrix} = \begin{bmatrix} T_D^{-1} \end{bmatrix} \begin{bmatrix} a_2 \\ b_2 \end{bmatrix}$$

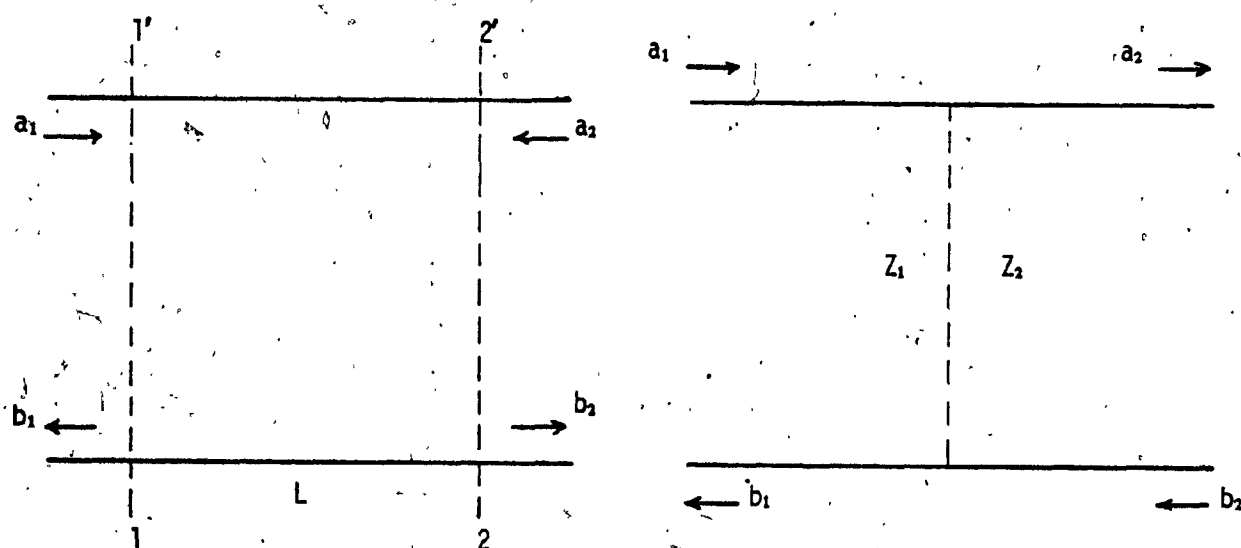


FIG 2.5
TRANSMISSION AND REFLECTION COEFFICIENTS
AT JUNCTION DISCONTINUITY

Comparing this with equation (2.5)

$$\begin{bmatrix} T_D^{-1} \end{bmatrix} = \frac{1}{2Z_2} \begin{bmatrix} Z_1 + Z_2 & Z_2 - Z_1 \\ Z_2 - Z_1 & Z_1 + Z_2 \end{bmatrix}$$

Now we have inverse transmission matrix of a uniform line of length θ and the inverse transmission matrix of discontinuity so resultant inverse transmission matrix of the discontinuity and the transmission line of length θ therefore is

$$\begin{bmatrix} T_D^{-1} & T_L^{-1} \end{bmatrix} = \frac{1}{2Z_2} \begin{bmatrix} Z_1 + Z_2 & Z_2 - Z_1 \\ Z_2 - Z_1 & Z_1 + Z_2 \end{bmatrix} \begin{bmatrix} e^{j\theta} & 0 \\ 0 & e^{-j\theta} \end{bmatrix}$$

$$= \begin{bmatrix} e^{j\theta} \cdot \frac{Z_1 + Z_2}{2Z_2} & e^{-j\theta} \cdot \frac{Z_2 - Z_1}{2Z_2} \\ e^{j\theta} \cdot \frac{Z_2 - Z_1}{2Z_2} & e^{-j\theta} \cdot \frac{Z_1 + Z_2}{2Z_2} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{e^{j\theta}}{2} + \frac{e^{j\theta} \cdot Z_1}{2Z_2} & \frac{e^{-j\theta}}{2} - \frac{Z_1 e^{-j\theta}}{2Z_2} \\ \frac{e^{j\theta}}{2} - \frac{Z_1 e^{j\theta}}{2Z_2} & \frac{e^{-j\theta}}{2} + \frac{Z_1 e^{-j\theta}}{2Z_2} \end{bmatrix}$$

$$= \frac{1}{2} \underbrace{\begin{bmatrix} e^{j\theta} & e^{-j\theta} \\ e^{j\theta} & e^{-j\theta} \end{bmatrix}}_A + \frac{1}{2} \cdot \frac{Z_1}{Z_2} \underbrace{\begin{bmatrix} e^{j\theta} & -e^{-j\theta} \\ -e^{j\theta} & e^{-j\theta} \end{bmatrix}}_B$$

$$= \frac{1}{2} \left[A + \frac{Z_1}{Z_2} B \right]$$

Now if there are n sections of transmission lines in cascade, each having the same length but different characteristic impedance, the resultant matrix can be written as

$$A_n = \frac{1}{2^{n+1}} \left[\prod_{i=1}^n \left(A + \frac{Z_i - 1}{Z_i} B \right) \right] (A_0 + \frac{Z_n}{R} B_0) \quad \dots (2.6)$$

where $A_0 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ and $B_0 = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

and R is the load impedance.

If $i = 1$, then we will get input impedance Z_0 of the transformer.

The matrix A_n is a 2×2 matrix where n is the number of sections of transmission lines in cascade.

The matrix A_n is a 2×2 matrix and can be represented by

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$$

The matrix element A_{11} represents the reciprocal of the amplitude of the transmission coefficient. Therefore the power absorbed by the load through the n section transmission line is

$$(A_{11} A_{11}^* R)^{-1}$$

where A_{11}^* is the complex conjugate of A_{11} and R is the load impedance of the end of the line and a unit amplitude of incident wave is assumed. If the Power Loss is defined as P_T , then

$$P_T = A_{11} A_{11}^* R$$

This is an even polynomial of $\cos\theta$ of degree $2n$.

The polynomial representing P_T can be written as

$$P_T = 1 + Q_{2n}(\cos\theta) \quad \text{where } Q_{2n}(\cos\theta) \text{ is an even polynomial of degree } 2n \text{ in } \cos\theta$$

2.3 Characteristics of Maximally Flat Transformer

A maximally flat transformer results when the n double zeros all coincide at the frequency f_0 when $\cos\theta=0$. Therefore the Power Loss Ratio can be written as

$$P_T = 1 + AK \cos^{2n}\theta$$

When the length of each section approaches 0, $\cos\theta = 1$ and the input impedance of the transformer is R , thus the Power Loss Ratio is

$P_T = (R+1)^2/4R$, since the amplitude transmission coefficient is $2R/(R+1)$ when $\cos\theta = 1$ therefore

$$1 + AK = \frac{(R+1)^2}{4R} \quad \text{therefore} \quad AK = \frac{(R-1)^2}{4R}$$

therefore

$$P_T = 1 + \frac{(R-1)^2}{4R} \cos^{2n}\theta$$

2.4 Design of Stepped Quarter Wave Transformer for the Case $n = 3$

The values for the characteristic impedances Z_i to give Maximally Flat Response in the Passband may be obtained by equating the coefficients of like powers of $\cos\theta$ in the two expressions.

$$P_T = A_{11} A_{11}^* R \quad P_T = 1 + AK \cos^{2n}\theta$$

and

$$A_n = \frac{1}{2^{n+1}} \left[\prod_{i=1}^n \left(A + \frac{Z_i - 1}{Z_i} B \right) \right] \left[A_0 + \frac{Z_n}{R} B_0 \right]$$

Since in our case $n = 3$, therefore

$$A = \frac{1}{16} \left[\left(A + \frac{Z_0}{Z_1} B \right) \left(A + \frac{Z_1}{Z_2} B \right) \left(A + \frac{Z_2}{Z_3} B \right) \right] \left[A_0 + \frac{Z_3}{R} B_0 \right]$$

$$A = \frac{1}{16} \left[A^3 + \frac{Z_2}{Z_3} A^2 B + \frac{Z_1}{Z_2} ABA + \frac{Z_1}{Z_3} AB^2 + \frac{Z_0}{Z_1} BA^2 + \frac{Z_0 Z_2}{Z_1 Z_3} BAB \dots \right. \\ \left. \dots + \frac{Z_0}{Z_2} B^2 A + \frac{Z_0}{Z_3} B^3 \right] \left[A_0 + \frac{Z_3}{R} B_0 \right]$$

Putting the values from Table 2.1 and simplifying

$$\begin{aligned} A = \frac{1}{16} \left[4 \cos^2 \theta \left(AA_0 + \frac{Z_3}{R} AB_0 + \frac{Z_0}{Z_3} BA_0 + \frac{Z_0}{R} BB_0 \right) + 2 \cos \theta \left(\frac{Z_2}{Z_3} + \frac{Z_1}{Z_3} \right) \dots \right. \\ \left. \dots \left(ABA_0 + \frac{Z_3}{R} ABB_0 \right) - 4 \sin^2 \theta \left(\frac{Z_1}{Z_2} AA_0 + \frac{Z_1 Z_3}{Z_2 R} AB_0 + \frac{Z_0 Z_2}{Z_1 Z_3} BA_0 \dots \right. \right. \\ \left. \left. + \frac{Z_0 Z_2}{Z_1 R} BB_0 \right) + 2 \cos \theta \left(\frac{Z_0}{Z_1} + \frac{Z_0}{Z_2} \right) \left(BAA_0 + \frac{Z_3}{R} BAB_0 \right) \right] \end{aligned}$$

Finding the values of A_{11} from table by using this expression of A and simplifying we get,

$$\begin{aligned} A_{11} = \frac{1}{2} \left[\left(1 + \frac{Z_0}{R} \right) \cos^3 \theta - \left\{ \frac{Z_1}{Z_0} + \frac{Z_2}{Z_3} + \frac{Z_1}{Z_2} + \frac{Z_0 Z_2}{Z_1 R} + \frac{Z_0 Z_2}{Z_1 R} + \frac{Z_0 Z_3}{Z_2 R} \right\} \sin^2 \theta \cos \theta \right. \\ \left. + j \left\{ \left(\frac{Z_3}{R} + \frac{Z_0}{Z_3} + \frac{Z_2}{R} + \frac{Z_1}{R} + \frac{Z_0}{Z_1} + \frac{Z_0}{Z_2} \right) \sin \theta \cos^3 \theta \dots \right. \right. \\ \left. \left. - \left(\frac{Z_1 Z_3}{Z_2 R} + \frac{Z_0 Z_2}{Z_1 Z_3} \right) \sin^3 \theta \right\} \right] \end{aligned}$$

Table 2.1

The reduction formulas which are used here are obtained by matrix multiplication:

$$A^n = (2\cos\theta)^{n-1}A$$

$$B^n = (2\cos\theta)^{n-1}B$$

$$A^r B^{n-r} = (2\cos\theta)^{n-2}AB$$

$$B^r A^{n-r} = (2\cos\theta)^{n-2}BA$$

$$A^s B^r A^{n-s-r} = (2\cos\theta)^{n-3}(-4\sin^2\theta)A$$

$$B^s A^r B^{n-s-r} = (2\cos\theta)^{n-3}(-4\sin^2\theta)B$$

Since we are interested in A_{11} only therefore the different terms which arise due to matrix multiplication are as follows:

A_{11}

$$AA_0 = 2\cos\theta$$

$$AB_0 = 2j\sin\theta$$

$$BA_0 = 2j\sin\theta$$

$$BB_0 = 2\cos\theta$$

$$ABB_0 = 4j\sin\theta\cos\theta$$

$$ABA_0 = -4\sin^2\theta$$

$$BAB_0 = -4\sin^2\theta$$

$$BAA_0 = 4j\sin\theta\cos\theta$$

Putting $Z_0 = 1$

And let

$$\frac{Z_1}{Z_3} + \frac{Z_2}{Z_3} + \frac{Z_1}{Z_2} + \frac{Z_2}{Z_1 R} + \frac{Z_3}{Z_1 R} + \frac{Z_3}{Z_2 R} = a$$

$$\frac{Z_3}{R} + \frac{1}{Z_3} + \frac{Z_2}{R} + \frac{Z_1}{R} + \frac{1}{Z_1} + \frac{1}{Z_2} = b$$

$$\frac{Z_1 Z_3}{Z_2 R} + \frac{Z_2}{Z_1 Z_3} = c$$

therefore

$$A_{11} = \frac{1}{2} \left[\left\{ \left(\frac{R+1}{R} \right) \cos^3 \theta - a(\sin^2 \theta \cos \theta) \right\} + j \{ b \sin \theta \cos^2 \theta - c \sin^3 \theta \} \right]$$

A_{11}^* = Complex Conjugate of A_{11}

$$A_{11}^* = \frac{1}{2} \left[\left\{ \left(\frac{R+1}{R} \right) \cos^3 \theta - a \sin^2 \theta \cos \theta \right\} - j \{ b \sin \theta \cos^2 \theta - c \sin^3 \theta \} \right]$$

therefore

$$A_{11} \times A_{11}^* = \frac{1}{4} \left[\left\{ \left(\frac{R+1}{R} \right) \cos^3 \theta - a \sin^2 \theta \cos \theta \right\}^2 + \{ b \sin \theta \cos^2 \theta - c \sin^3 \theta \}^2 \right]$$

Simplifying this expression,

$$A_{11} \times A_{11}^* = \frac{1}{4} \left[\left\{ \left(\frac{R+1}{R} \right)^2 - c^2 - b^2 + \frac{2a(R+1)}{R} + a^2 - 2bc \right\} \cos^6 \theta \dots \right]$$

$$\begin{aligned} & \dots + \left\{ 3c^2 + b^2 - \frac{2a(R+1)}{R} - 2a^2 + 4bc \right\} \cos^4 \theta \dots \\ & \dots + \left\{ -3c^2 + a^2 - 2bc \right\} \cos^2 \theta + c^2 \end{aligned} \quad \left. \vphantom{\begin{aligned} & \dots + \left\{ 3c^2 + b^2 - \frac{2a(R+1)}{R} - 2a^2 + 4bc \right\} \cos^4 \theta \dots \\ & \dots + \left\{ -3c^2 + a^2 - 2bc \right\} \cos^2 \theta + c^2 \end{aligned}} \right] \quad \left. \vphantom{\begin{aligned} & \dots + \left\{ 3c^2 + b^2 - \frac{2a(R+1)}{R} - 2a^2 + 4bc \right\} \cos^4 \theta \dots \\ & \dots + \left\{ -3c^2 + a^2 - 2bc \right\} \cos^2 \theta + c^2 \end{aligned}} \right]$$

$$\text{Now } P_T = A_{11} A_{11}^* R$$

$$A_{11} A_{11}^* R = \frac{R}{4} \left\{ \frac{(R+1)^2}{R^2} - c^2 - b^2 + \frac{2a(R+1)}{R} + a^2 - 2bc \right\} \cos^6 \theta \dots$$

$$- \frac{R}{4} \left\{ -3c^2 - b^2 + \frac{2a(R+1)}{R} + 2a^2 - 4bc \right\} \cos^4 \theta \dots$$

$$+ \frac{R}{4} \left\{ -3c^2 + a^2 - 2bc \right\} \cos^2 \theta$$

$$+ \frac{R}{4} c^2 \quad \dots (2.7)$$

Now P_T is also equal to

$$P_T = 1 + AK \cos^{2n} \theta$$

where

$$AK = \frac{(R-1)^2}{4R}$$

Since $R = 5$

$$\text{therefore } AK = \frac{16}{20} = 0.8$$

therefore

$$P_T = 1 + 0.8 \cos^6 \theta \quad \dots (2.8)$$

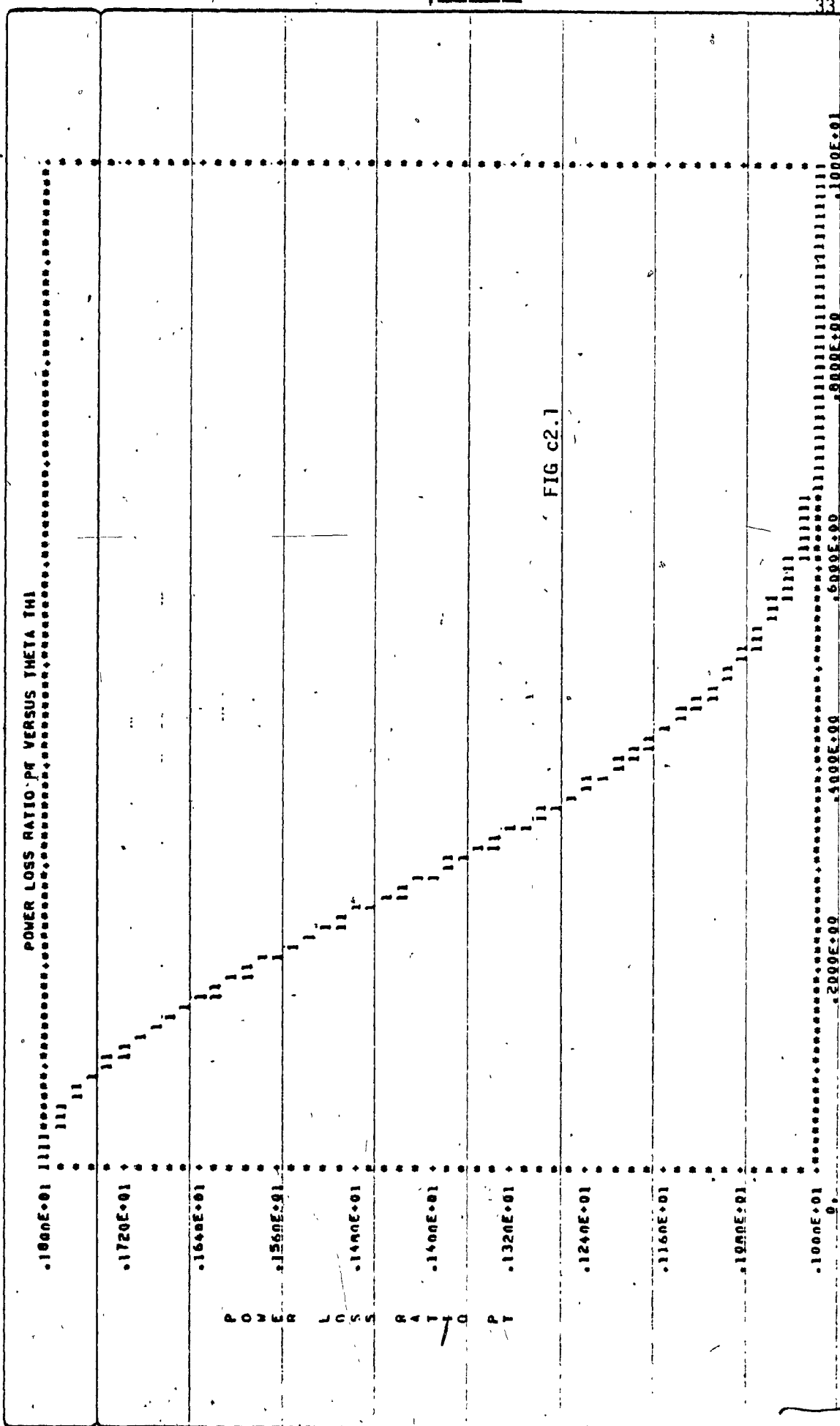
A computer program (Table 2.2) is written using plotting subroutine to plot the characteristics (Fig. c2.1) of P_T versus θ . Comparing the

Table 2.2

```

C      PROGRAM FLAT(INPUT,OUTPUT)
C      THIS PROGRAM PLOTS THE POWER LOSS RATIO PT FOR
C      THE THIRD ORDER BUTTERWORTH AGAINST
C      ELECTRICAL LENGTH THETA WHICH IS NORMALIZED TO
C      TH1 WHERE TH1=TH/1.6, AND THETA IS A FUNCTION OF
C      FREQUENCY.
      DIMENSION X(161),Y(161,1),A(160),IMAG4(5151)
      PRINT 5
5      FORMAT (1H1,//////,21X,*TH1*,23X,*PT*)
      DO 15 I=1,161
      TH=(I-1.0)*0.01
      R=5.0
      AK=(R-1)**2/(4.0*R)
      Q=(COS(TH))**6
      PT=1.0+AK*Q
      TH1=TH/1.6
      X(I)=TH1
      Y(I,1)=PT
      PRINT 10,TH1,PT
10     FORMAT (//,15X,E15.8,10X,E15.8)
15     CONTINUE
      READ 20, (A(I),I=1,160)
20     FORMAT (80A1)
      CALL USPLH(X,Y,161,1,1,161,A,IMAG4,IER)
      STOP
      END

```



coefficients of equations (2.7) and (2.8) for equal powers of $\cos\theta$.

Comparing constant terms

$$\frac{R}{4} c^2 = 1 \quad \text{or } c^2 = \frac{4}{R} \quad \text{or } c = \frac{2}{\sqrt{R}}$$

therefore

$$\frac{Z_1 Z_3}{Z_2 R} + \frac{Z_2}{Z_1 Z_3} = \frac{2}{\sqrt{R}} \quad \dots (2.9)$$

Now for matching the impedances with load, we use the following symmetry relation

$$Z_i Z_{n+1-i} = R$$

since $n = 3$

therefore

$$Z_1 Z_3 = R$$

If we put this relation in equation (2.9)

$$\frac{R}{Z_2 R} + \frac{Z_2}{R} = \frac{2}{\sqrt{R}}$$

therefore

$$Z_2 = \sqrt{R} \quad \text{or } Z_2^2 = R$$

$$\therefore Z_1 Z_3 = R$$

$$\text{and } Z_2^2 = R$$

Comparing the coefficients of $\cos^2\theta$

$$\frac{R}{4} \left\{ \frac{(R+1)^2}{R^2} - c^2 - b^2 + \frac{2a(R+1)}{R} + a^2 - 2bc \right\} = 0.8$$

therefore

$$\frac{(R+1)}{2} \left\{ \frac{(R+1)}{2R} + a \right\} + \frac{R}{4} \{ a^2 - b^2 - 2bc \} = 1.8$$

Now $Z_1 Z_3 = R$ and $Z_2^2 = R$ where $R = 5$

$$a = (0.2Z_1^4 + 0.8944272Z_1^3 + 0.8944272Z_1 + 1)/Z_1^2$$

$$b = (2 + 0.4Z_1^2 + 0.8944272Z_1)/Z_1$$

So putting the values of a and $(a^2 - b^2 - 2bc)$ in the above equation we get

$$\begin{aligned} Z_1^8 + 8.94427Z_1^7 + 28Z_1^6 + 26.832822Z_1^5 - 50Z_1^4 - 80.498448Z_1^3 \dots \\ \dots - 20Z_1^2 + 44.72136Z_1 + 25 = 0 \end{aligned} \quad \dots(2.10)$$

Comparing the coefficients of $\cos^4 \theta$

$$\frac{R}{4} \{-3c^2 - b^2 + \frac{2a(R+1)}{R} + 2a^2 - 4bc\} = 0$$

Putting the values of a , b , c and a^2 , b^2 , simplifying we get

$$\begin{aligned} Z_1^8 + 8.94427Z_1^7 + 24Z_1^6 + 8.944276Z_1^5 - 50Z_1^4 - 62.609904Z_1^3 \dots \\ \dots + 0Z_1^2 + 44.72136Z_1 + 25 = 0 \end{aligned} \quad \dots(2.11)$$

Comparing the coefficient of $\cos^2 \theta$

$$\frac{R}{4} \{-3c^2 + a^2 - 2bc\} = 0$$

Putting the values of a , b , c and R and simplifying

$$\begin{aligned} Z_1^8 + 8.94427Z_1^7 + 20Z_1^6 - 8.94427Z_1^5 - 50Z_1^4 - 44.72136Z_1^3 \dots \\ \dots + 20Z_1^2 + 44.72136Z_1 + 25 = 0 \end{aligned} \quad \dots(2.12)$$

Now we have three simultaneous equations (2.10), (2.11) and (2.12). These three equations are of order eight and the solution of these 8th order equations is not possible in ordinary way. We want one real root common in all these equations. So to find the roots of these equations a root finding computer subroutine is used which gives the zeros of these equations.

Table 2.3

PROGRAM ZERO5	73/172	OPT-1	FTN 4.7+470	79/08/30, 11.30.59	PAGE
1	PROGRAM ZERO5(INPUT,OUTPUT)				1
	COMPLEX Z				
	DIMENSION A(9),Z(8)				
	NDEG=8				
5	READ*,(A(I),I=1,9)				
	CALL ZRPOLY (A,NDEG,Z,IER)				
	PRINT 5				
5	FORMAT(7(/))				
10	PRINT*,(Z(I),I=1,8)				
	STOP				
	END				
	(.8382310466968,0.) (-.4248760080109,0.) (-.7023625974441,.6762712209763) (-.7023625974441,-.6762712209763)				
	(1.225239645906,0.) (-2.341746541842,2.007879377205) (-2.341746541842,-2.007879377205) (-4.29264642602,0.)				
	(1.000000023873,0.) (-.7023625760427,.6762712259264) (-.7023625760427,-.6762712259264) (-.99999998363732,0.)				
	(1.225239645906,0.) (-2.238043874794,0.) (-2.234098842735,0.) (-4.292641954349,0.)				
	(1.225145486221,0.) (-.7024885468424,.6762446117631) (-.7024885468424,-.6762446117631) (-.702236809749,.676397792217)				
	(-.702236809749,.676397792217) (1.225313902279,0.) (-4.292649337659,.002991693744429) (-4.292649337659,-.002991693744429)				

After looking at the solution of these equations from a computer program Table (2.3) it is seen that there is one real root which satisfies all the equations so selecting this root as value of Z_1 .

$$\therefore Z_1 = 1.225239656$$

$$Z_2 = \sqrt{R} = 2.2360679$$

and $Z_3 = R/Z_1 = 4.0808344$

$$Z_1 = 1.225239656$$

$$Z_2 = 2.2360679$$

$$Z_3 = 4.0808344$$

BUTTERWORTH CASE

Now to show the plot with these values of characteristic impedances we use the Power Loss Equation (2.7) in a plotting subroutine which plots the Butterworth characteristics using the above values of characteristic impedances, (Table 2.4, Fig. c2.2). This characteristic is exactly the same as compared to the one plotted using equation (2.8) without using the values of characteristic impedances, (Fig. c2.1).

Now the values of these characteristic impedances are changed slightly and the effect is observed. It is seen that it does not make any changes in the behaviour of the curve, (Table 2.5, Fig. c2.3).

2.5 Characteristics of Chebyshev Transformer

Power Loss Ratio is

$$P_T = A_{11} A_{11}^* R \quad \text{which is an even polynomial of } \cos\theta \text{ of degree}$$

Table 2.4

```

PROGRAM CHAR (INPUT,OUTPUT)
C   THIS PROGRAM PLOTS THE POWER LOSS RATIO PT FOR THE THIRD ORDER
C   BUTTERWORTH AGAINST ELECTRICAL LENGTH THETA WHICH IS NORMALIZED
C   TO TH1
C   IN THIS PROGRAM THE CHARACTERISTIC IMPEDANCES WHICH ARE DETERMINED
C   FOR BUTTERWORTH CASE ARE USED IN POWER LOSS RATIO PT.
DIMENSION X(161),Y(161,1),A(160),IMAG4(5151)
PRINT 5
5  FORMAT (1H1,////,21X,TH1*,23X,PT*,//)
DO 15 I=1,161
TH=(I-1.0)*0.01
R=5.0
Z1=1.225239656
Z2=2.2340679
Z3=4.0808344
A2=Z1/Z3+Z2/Z3+Z1/Z2+Z2/(Z1*R)+Z3/(Z1*R)+Z3/(Z2*R)
B=Z3/R+1/Z3+Z2/R+Z1/R+1/Z1+1/Z2
C=(Z1*Z3)/(Z2*R)+Z2/(Z1*Z3)
Q=(COS(TH))**6
P=(COS(TH))**4
Q=(COS(TH))**2
PT=(R/4*((R+1)**2/(R**2))-C**2-B**2+(2*A2*(R+1))/
1R+A2**2-2*B*C))*Q-(R/4*(-3*C**2-B**2+(2*A2*(R+1))/
1R+2*A2**2-4*B*C))*P
*+(R/4*(-3*C**2+A2**2-2*B*C))*Q
*+(R/4)*C**2
TH1=TH/1.6
X(I)=TH1
Y(I,1)=PT
PRINT 10,TH1,PT
10  FORMAT (15X,E15.8,10X,E15.8)
15  CONTINUE
READ 20,(A(I),I=1,160)
20  FORMAT (80A1)
CALL USPLH (X,Y,161,1,1,161,A,IMAG4,IER)
STOP
END

```

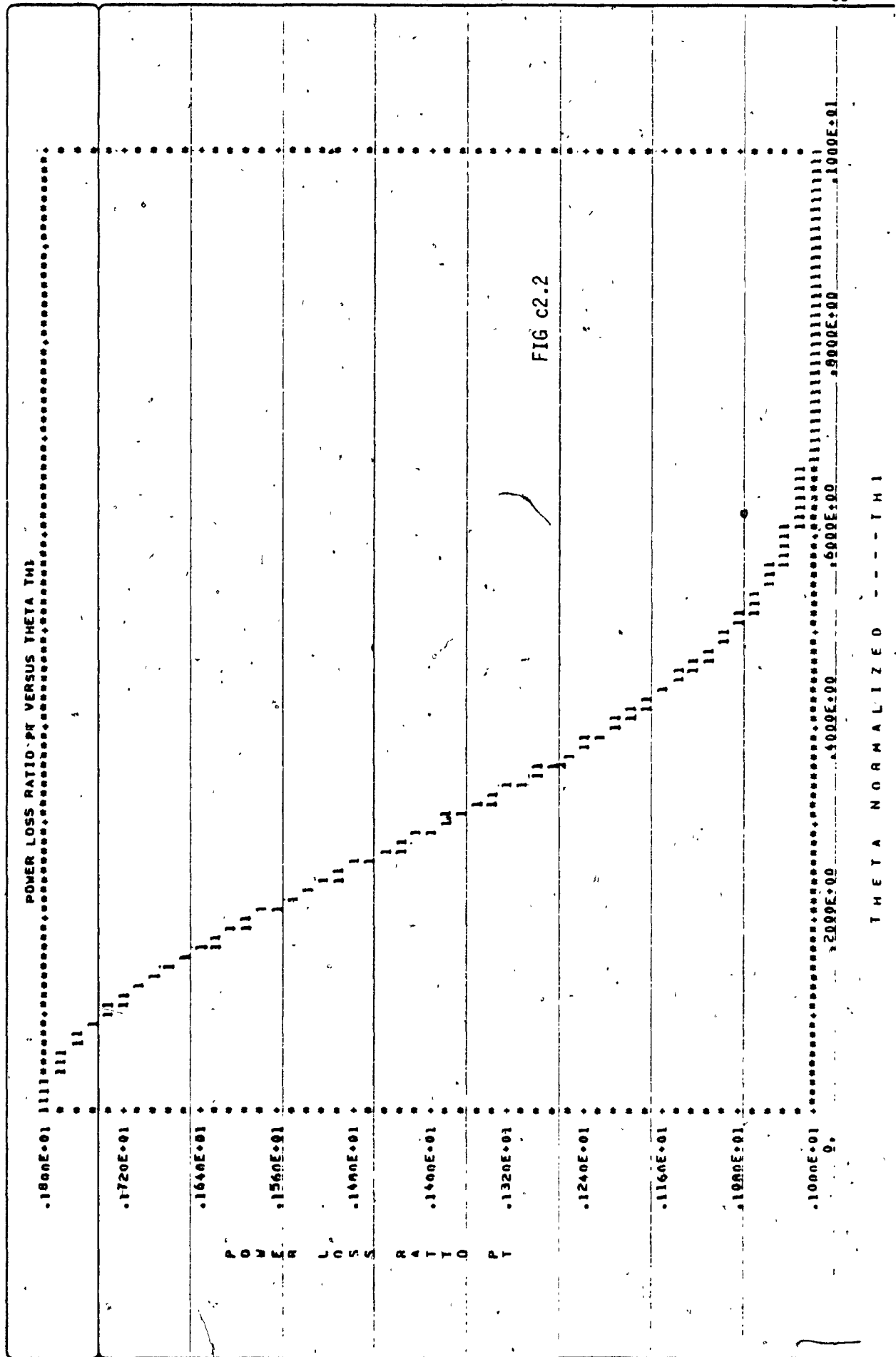
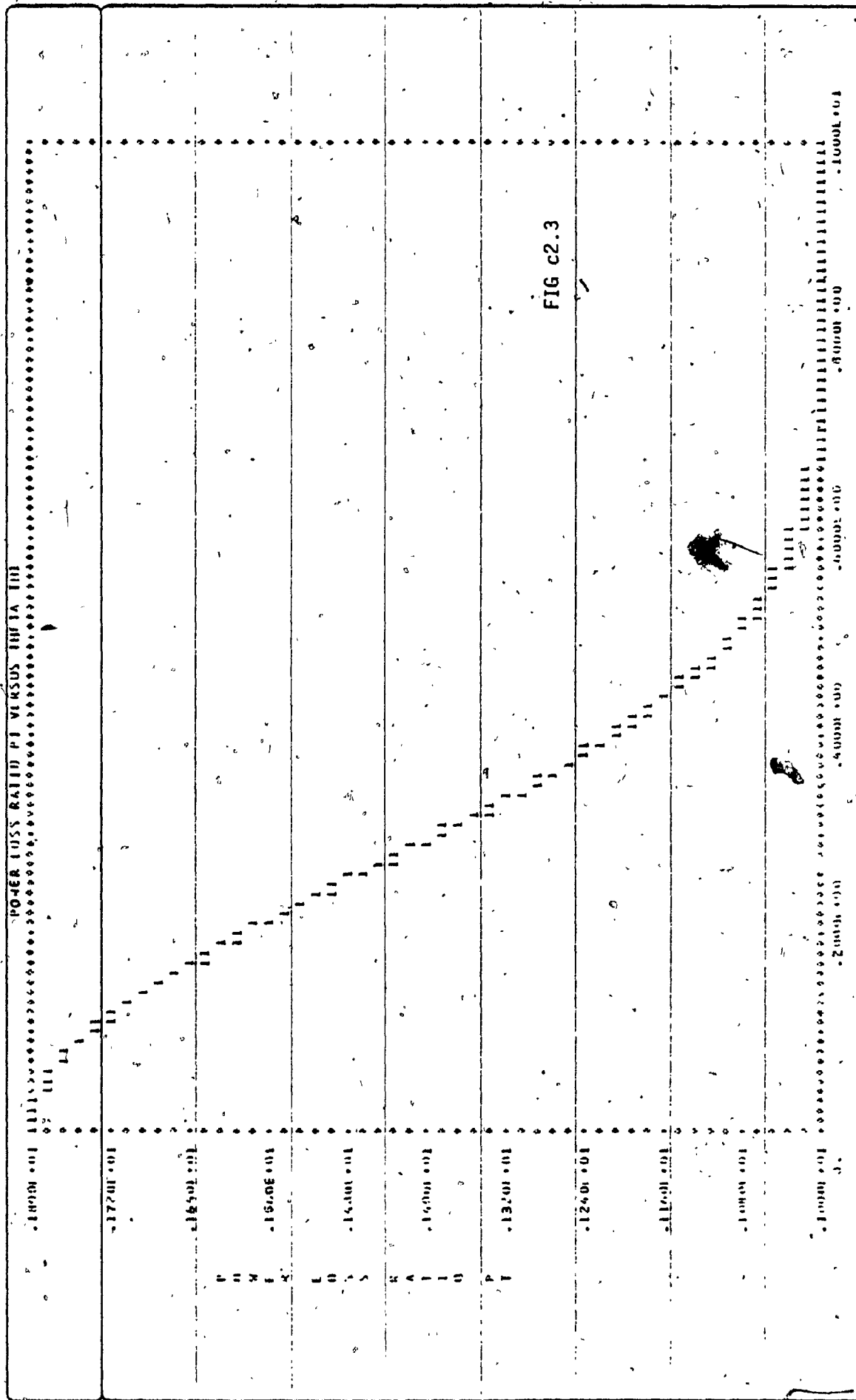


Table 2.5

```

PROGRAM CHAR (INPUT,OUTPUT)
C   THIS PROGRAM PLOTS THE POWER LOSS RATIO PT FOR THE THIRD ORDER
C   BUTTERWORTH AGAINST ELECTRICAL LENGTH THETA WHICH IS NORMALIZED
C   TO TH1.
C   IN THIS PROGRAM THE CHARACTERISTIC IMPEDANCES WHICH ARE DETERMINED
C   FOR BUTTERWORTH CASE ARE CHANGED SLIGHTLY AND USED IN POWER
C   LOSS RATIO PT.
C   DIMENSION X(161),Y(161,1),A(160),IMAG4(5151)
C   PRINT 5
5   FORMAT (1H1,////,21X,*TH1*,23X,*PT*,//)
C   DO 15 I=1,161
C   TH=(I-1.0)*0.01
C   R=5.0
C   Z1=1.325239656
C   Z2=2.3360679
C   Z3=4.1808344
C   A2=Z1/Z3+Z2/Z3+Z1/Z2+Z2/(Z1*R)+Z3/(Z1*R)+Z3/(Z2*R)
C   B=Z3/R+1/Z3+Z2/R+Z1/R+1/Z1+1/Z2
C   C=(Z1*Z3)/(Z2*R)+Z2/(Z1*Z3)
C   Q=(COS(TH))**6
C   P=(COS(TH))**4
C   Q=(COS(TH))**2
C   PT=(R/4*((R+1)**2/(R**2))-C**2-B**2+(2*A2*(R+1))/
10  1R+A2**2-2*B*C))*Q-(R/4*(-3*C**2-B**2+(2*A2*(R+1))/
15  1R+2*A2**2-4*B*C))*P
C   S=(R/4*(-3*C**2+A2**2-2*B*C))*Q
C   S=(R/4)*C**2
C   TH1=TH/1.6
C   X(I)=TH1
C   Y(I,1)=PT
C   PRINT 10,TH1,PT
10  FORMAT (15X,E15.8,10X,E15.8)
15  CONTINUE
C   READ 20,(A(I),I=1,160)
20  FORMAT (80A1)
C   CALL USPLH (X,Y,161,1,1,161,A,IMAG4,IER)
C   STOP
C   END

```



INITIAL IBI

2n. P_T can also be written as

$P_T = 1 + Q_{2n}(\cos\theta)$ where $Q_{2n}(\cos\theta)$ is an even polynomial of degree 2n in $\cos\theta$.

By looking at the above polynomial it is seen that optimum performance can be obtained if the polynomial Q_{2n} has n double zeros in the passband and is such that power loss ratio reaches the same maximum between zeros in the passband.

If Chebyshev Polynomial is selected it will give an optimum performance because Power Loss ratio will reach the same maximum between zeros due to the ripples of Chebyshev polynomial. Therefore synthesis of P_T can be carried so that the desired bandwidth and the passband tolerance needed for the transformer can be achieved.

A synthesis of P_T can be achieved by choosing a Chebyshev Polynomial of suitable degree.

$$\therefore P_T = 1 + AK T_n^2\left(\frac{\cos\theta}{p}\right)$$

where

$T_n\left(\frac{\cos\theta}{p}\right)$ is the Chebyshev polynomial of degree n and AK and p are arbitrary scale factors which will determine the bandwidth of passband tolerance of P_T .

When the length of each section approaches 0, $\cos\theta = 1$ and the input impedance of the transformer is R, thus the Power Loss Ratio is

$$P_T = \frac{(R+1)^2}{4R}, \text{ since the amplitude transmission coefficient is } \frac{2R}{R+1} \text{ when } \cos\theta = 1$$

$$\therefore P_T = \frac{(R+1)^2}{4R} = 1 + AK T_n^2(1/p)$$

$$\therefore \frac{(R+1)^2}{4R} = 1 + AK T_n^2(1/p) \quad \dots(2.13)$$

The bandwidth and passband tolerance of a n -section transformer are determined by the properties of the Chebyshev polynomial of degree n and the load impedance R without the knowledge of impedances Z_i .

The zeros of $T_n(\frac{\cos\theta}{p})$ occur when $\cos\theta = px_s = \cos\theta_s$ and $s = 1, 2, \dots, n$, where x_s are zeros of $T_n(x)$.

The scale factor p is given by

$p = \frac{\cos\theta_s}{x_s}$ where s corresponds to anyone particular zero of $T_n(x)$.

AK in equation (2.13) is the Passband Tolerance, so solving equation (2.13) for AK

$$\frac{(R+1)^2}{4R} = 1 + AK T_n^2(1/p)$$

$$AK = \frac{(R-1)^2}{4R T_n^2(1/p)}$$

Now if scale factor p is chosen, the passband tolerance AK is fixed and vice versa so we have to choose either one of them. If p were taken equal to unity then passband would cover a 200 percent frequency band and the tolerance in the passband would be $(R-1)^2/4R$ which would give a Power Loss Ratio equal to that existing when there

is no matching transformer. For any practical case, therefore p will always be less than unity.

Choosing a Chebyshev Polynomial of third order

$$T_3(Z) = 4Z^3 - 3Z$$

$$\therefore T_n^2(1/p) = T_3^2(1/p) = [4(1/p)^3 - 3(1/p)]^2$$

Selecting $R = 5$

$$\therefore AK = \frac{16}{20T_3^2(1/p)} = \frac{0.8}{T_3^2(1/p)}$$

Since scale factor should be less than unity so for different values of p from 0.85 to 1.0 the Passband Tolerance AK is calculated by writing a computer program (Table 2.6).

The final expression for the Power Loss Ratio is

$$P_T = 1 + AK T_3^2(\cos\theta/p)$$

So a computer program using plotting subroutine is written to plot the characteristics of P_T versus θ for different values of p and AK for third order Chebyshev Polynomial i.e., $n = 3$ (Table 2.7, Figs. c2.4, c2.5, c2.6, c2.7, c2.8, c2.9). The Power Loss Ratio Polynomial has the following properties;

1. It is equal to unity at n values of θ given by $\cos\theta = (\cos\theta_s)x_i/x_s$, $i = 1, 2, \dots, n$ where x_i is a zero of $T_n(x)$. At these points all the incident power is transmitted to the load.
2. Between the zeros P_T has a maximum value of

$$P_T = \frac{1 + (R - 1)^2}{4R T_n^2(1/p)}$$

Table 2.6

```
PROGRAM PB(INPUT,OUTPUT)
C THIS PROGRAM CALCULATES THE VALUE OF
C PASSBAND TOLERANCE AK FOR DIFFERENT VALUES
C OF SCALE FACTOR P FROM 0.85 TO 1.00
REAL AK,T3,P,Q
P=0.84
PRINT 5
4 I=1
P=P+(I*0.01)
Q=1/P
T3=(4*Q**3-3*Q)**2
AK=0.8/T3
PRINT 6,P,AK
5 FORMAT(1H1,/////,25X,*P*,25X,*AK*)
6 FORMAT(/,23X,F4.2,15X,E20.14)
IF (P.LE.0.99) GO TO 4
END
```


Table 2.7

```

C      PROGRAM PLOT(INPUT,OUTPUT)
C      THIS PROGRAM PLOTS THE POWER LOSS RATIO PT FOR
C      THE THIRD ORDER CHEBYSHEV POLYNOMIAL AGAINST
C      ELECTRICAL LENGTH THETA WHICH IS NORMALIZED TO
C      TH1. WHERE TH1=TH/1.6, AND THETA IS A FUNCTION OF
C      FREQUENCY.
C      THE POWER LOSS RATIO IS PLOTTED FOR DIFFERENT
C      VALUES OF SCALE FACTOR P AND PASSBAND
C      TOLERANCE AK.
      DIMENSION X(161),Y(161,1),A(160),IMAG4(5151)
      READ*,P,AK
      PRINT 5,P,AK
5      FORMAT (1H1,////,10X,*SCALE FACTOR P=*,E15.8,
*10X,*PASSBAND TOLERANCE AK=*,E15.8)
      PRINT 10
10     FORMAT(///,16X,*TH1*,22X,*PT*,//)
      DO 30 I=1,161
      TH=(I-1.0)*0.01
      Q=COS(TH)/P
      T1=(4.0*Q*Q*Q-3.0*Q)
      T3=T1*T1
      PT=1.0+AK*T3
      TH1=TH/1.6
      X(I)=TH1
      Y(I,1)=PT
      PRINT 20,TH1,PT
20     FORMAT(9X,E15.8,10X,E15.8)
30     CONTINUE
      READ 40, (A(I),I=1,160)
40     FORMAT (80A1)
      CALL USPLH(X,Y,161,1,1,161,A,IMAG4,IER)
      STOP
      END

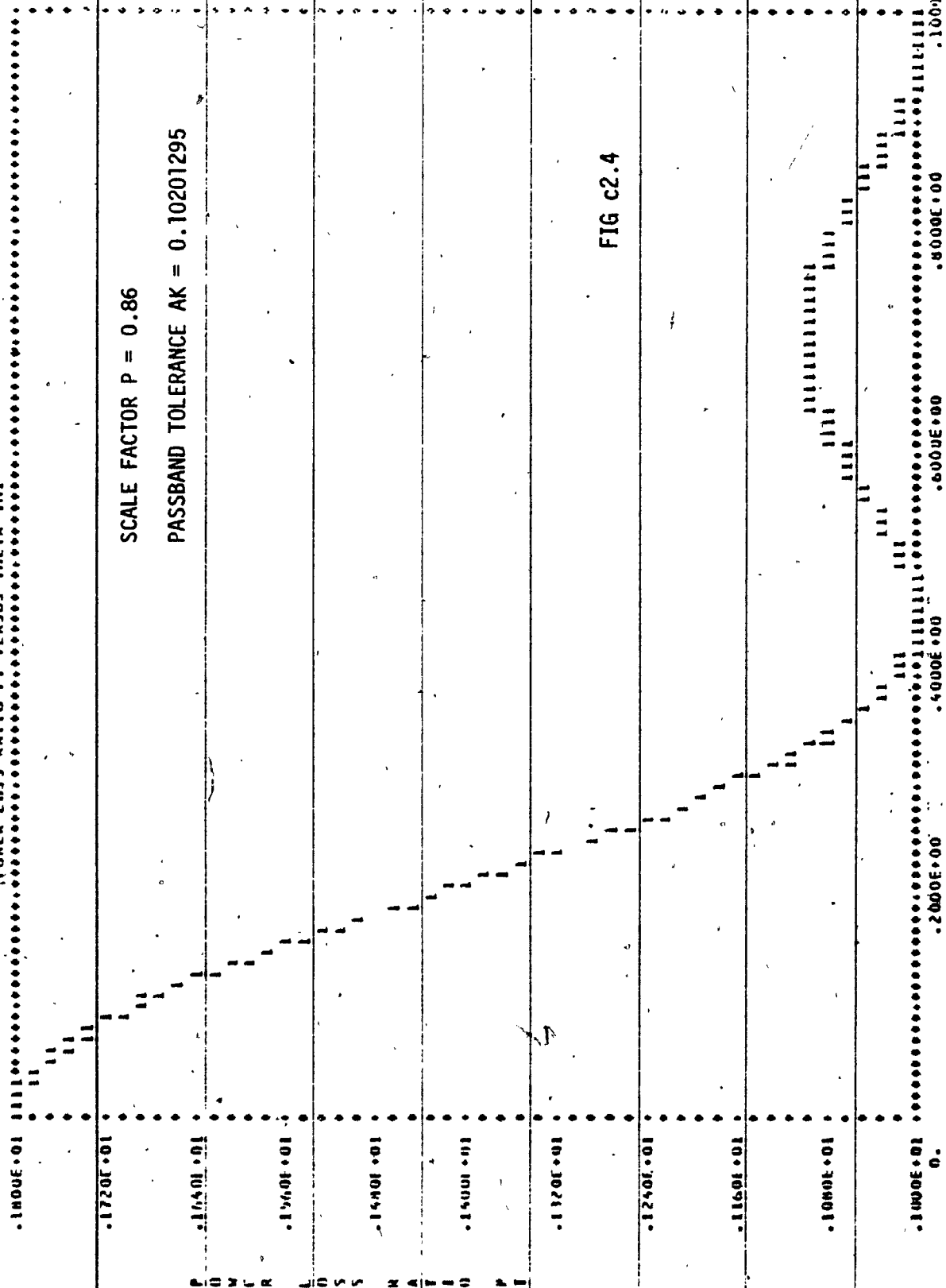
```

POWER LOSS RATIO PL VERSUS THETA TH1

SCALE FACTOR P = 0.86

PASSBAND TOLERANCE AK = 0.10201295

FIG C2.4



THETA NORMALIZED - - - TH1

POWER LOSS RATIO PL VERSUS THETA TH



SCALE FACTOR P = 0.87
PASSBAND TOLERANCE AK = 0.11600182

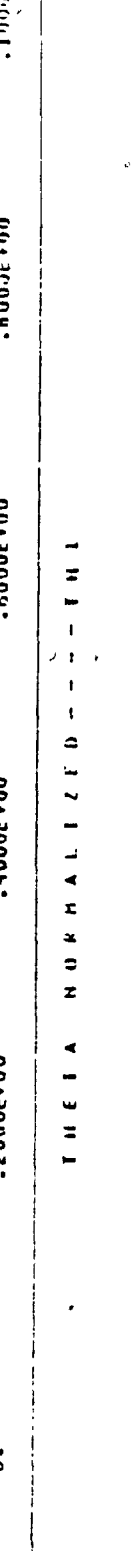
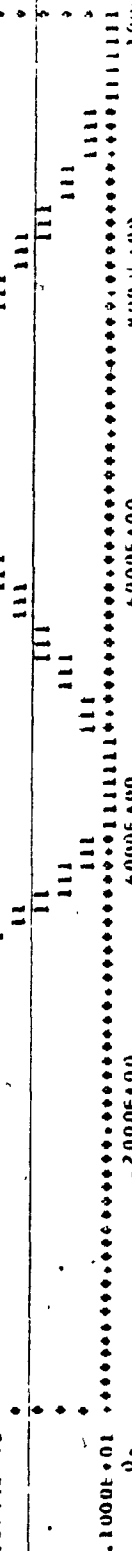
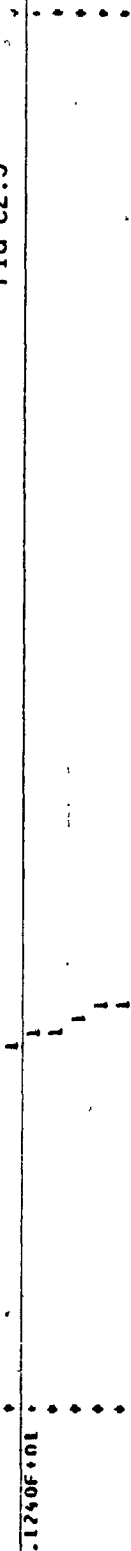
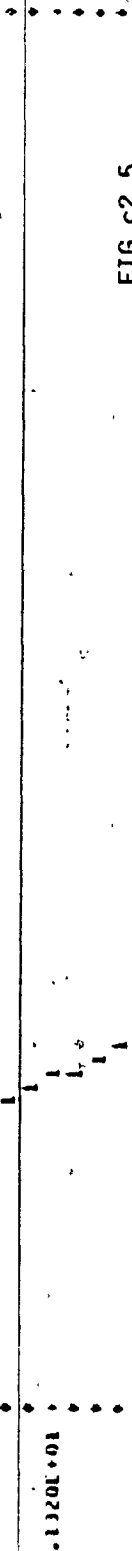
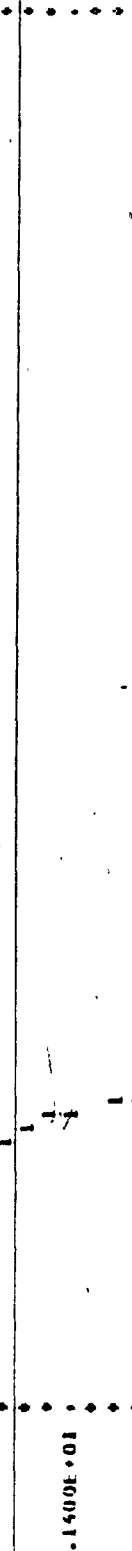
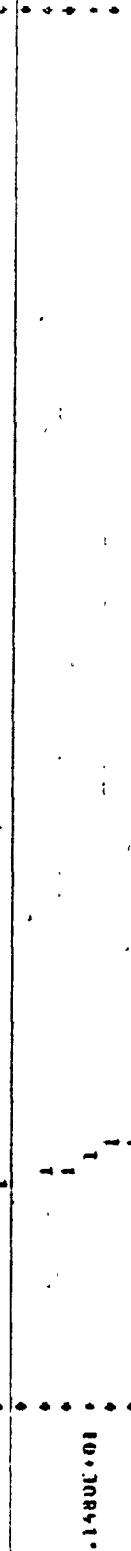
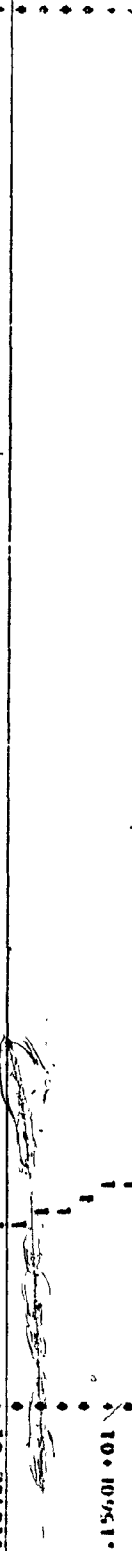
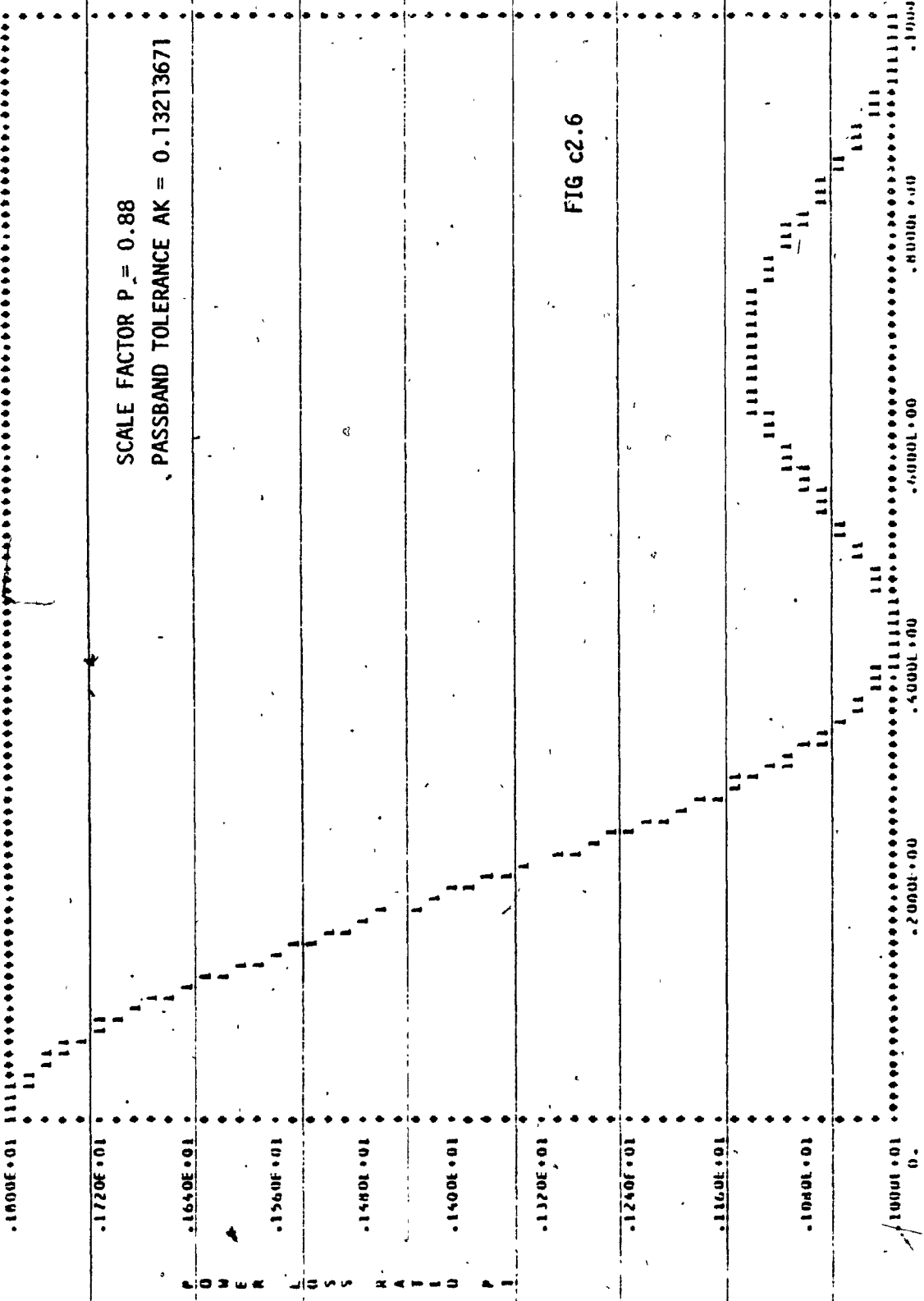


FIG c2.5

POWER LOSS RATIO PL VERSUS THETA TH

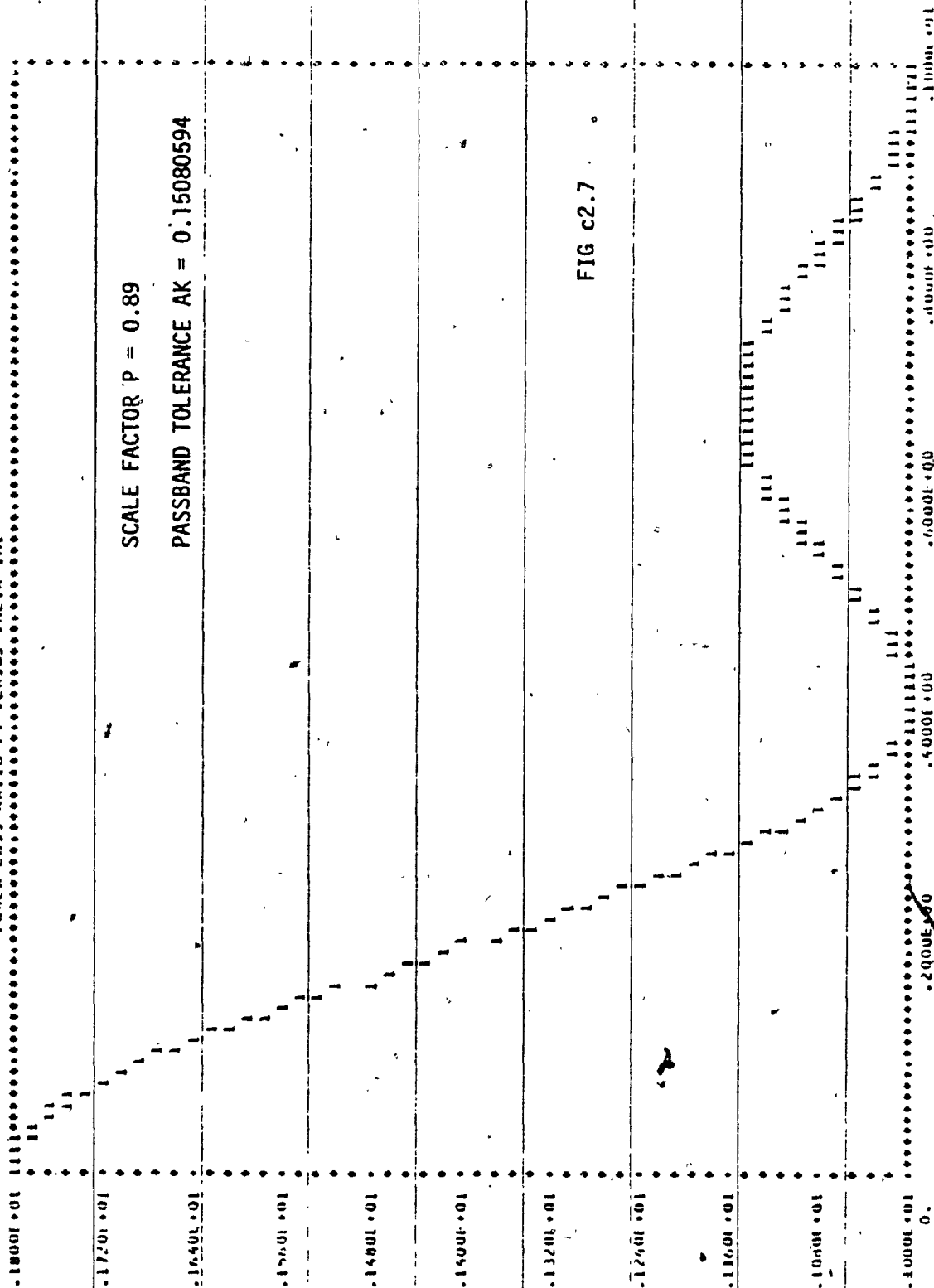


THETA NORMALIZED - - - - - TH

POWER LOSS RATIO PL VERSUS THETA 101

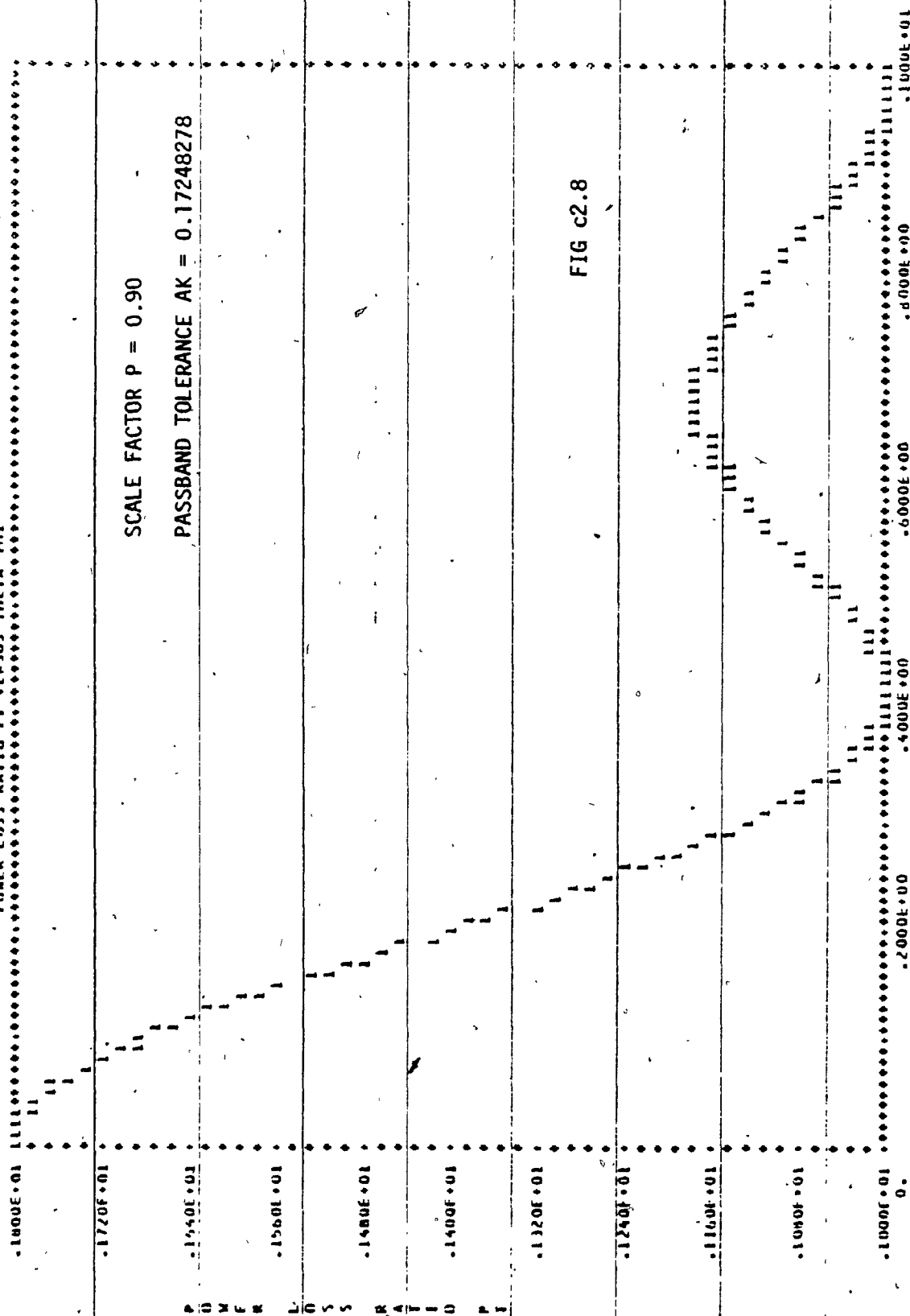
SCALE FACTOR P = 0.89
PASSBAND TOLERANCE AK = 0.15080594

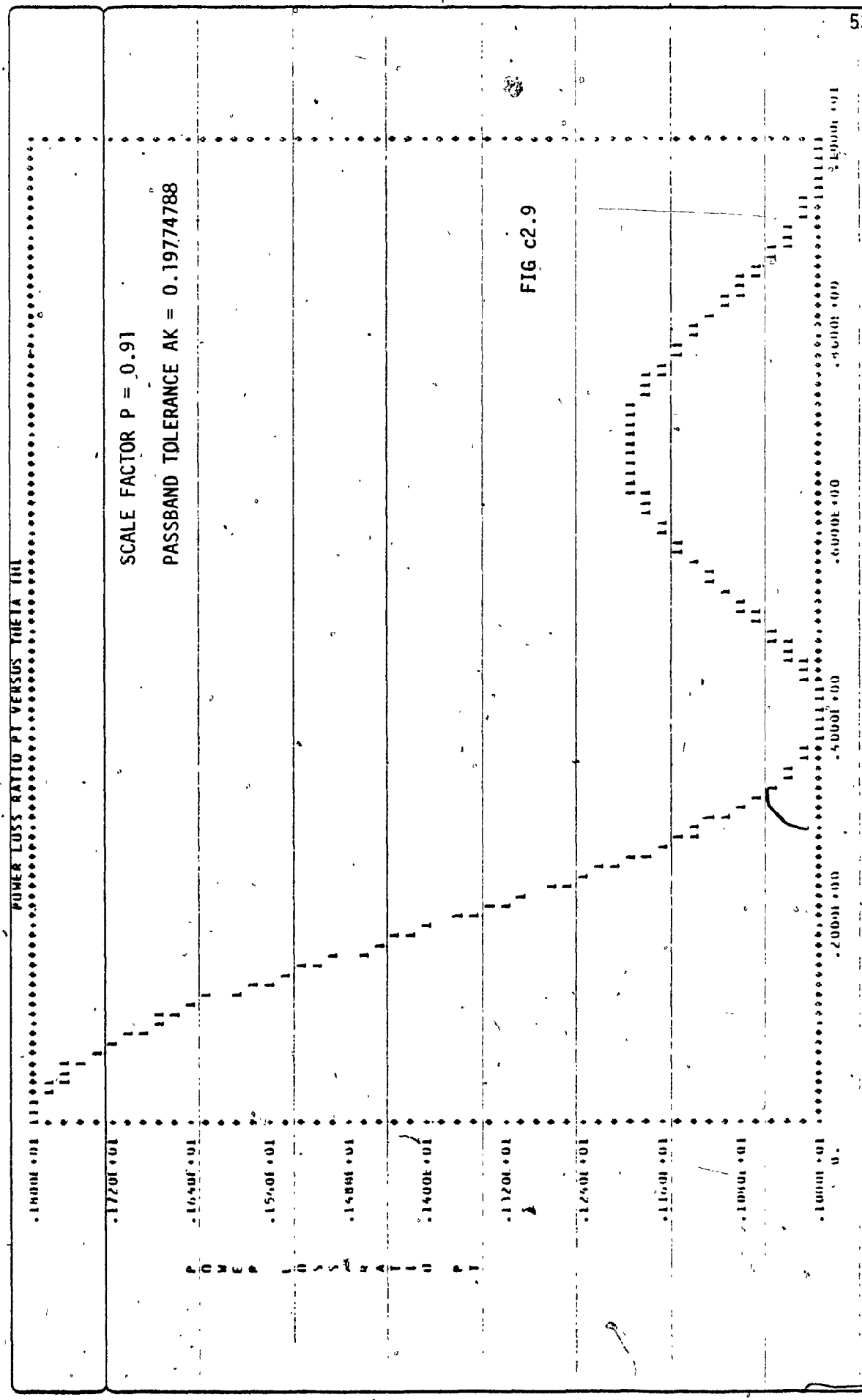
FIG c2.7



THETA NORMALIZED TO 1

POWER LOSS RATIO PL VERSUS FREQUENCY





DATA NORMALIZED

3. No other Polynomial with the same passband tolerance can have a greater bandwidth, since if it did, it would imply that the Chebyshev transformer of the same bandwidth would have a greater tolerance in the passband and this is not possible because if it was possible then it would have cross the Chebyshev Polynomial. Therefore a transformer designed to give Chebyshev behaviour in the passband is an optimum design as far as maximum bandwidth and minimum passband tolerance are concerned.

2.6 Design of Stepped Quarter Wave Transformer for the Case $n=3$

The values for the characteristic impedances Z_i to give Chebyshev behaviour in the passband may be obtained by equating the coefficients of like powers of $\cos\theta$ in the two expressions which are as follows:

$$P_T = A_{11} A_{11}^* R$$

and

$$P_T = 1 + AK T_n^2 \left[\cos\theta/P \right]$$

$$\begin{aligned} \text{Now } P_T = A_{11} A_{11}^* R &= \frac{R}{4} \left\{ \frac{(R+1)^2}{R^2} - c^2 - b^2 + \frac{2a(R+1)}{R} + a^2 - 2bc \right\} \cos^4 \theta \\ &\quad - \frac{R}{4} \left\{ -3c^2 - b^2 + \frac{2a(R+1)}{R} + 2a^2 - 4bc \right\} \cos^2 \theta \\ &\quad + \frac{R}{4} \left\{ -3c^2 + a^2 - 2bc \right\} \cos^2 \theta \\ &\quad + \frac{R}{4} c^2 \end{aligned} \quad \dots (2.14)$$

This equation has been derived for Butterworth case.

Now P_T is also equal to

$$P_T = 1 + Ak T_n^2 [\cos \theta / P]$$

since $n = 3 \therefore T_3^2(Z) = (4Z^3 - 3Z)^2 = 16Z^6 + 9Z^4 - 24Z^2$

$$\therefore T_3^2 [\cos \theta / P] = \frac{16 \cos^6 \theta}{P^6} - \frac{24 \cos^4 \theta}{P^4} + \frac{9 \cos^2 \theta}{P^2}$$

Now $AK = \text{Passband Tolerance}$ and $P = \text{Scale Factor}$.

Selecting $P = 0.91$, corresponding Passband Tolerance $AK = 0.1977479$

$$\therefore P_T = 1 + 0.1977479 \left[\frac{16 \cos^6 \theta}{P^6} - \frac{24 \cos^4 \theta}{P^4} + \frac{9 \cos^2 \theta}{P^2} \right]$$

$$\therefore P_T = 5.5716483 \cos^6 \theta - 6.920826 \cos^4 \theta + 2.149174 \cos^2 \theta + 1 \dots$$

...(2.15)

Comparing the coefficients of equations (2.14) and (2.15) for equal powers of $\cos \theta$.

Comparing constant terms.

$$\frac{R}{4} c^2 = 1 \quad \text{or} \quad c^2 = \frac{4}{R} \quad \text{or} \quad c = \frac{2}{\sqrt{R}}$$

$$\left(\frac{Z_1 Z_3}{Z_2 R} + \frac{Z_2}{Z_1 Z_3} \right) = \frac{2}{\sqrt{R}} \quad \dots(2.16)$$

Comparing the coefficients of $\cos^6 \theta$

$$\frac{R}{4} \left\{ \frac{(R+1)^2}{R^2} - c^2 - b^2 + \frac{2a(R+1)}{R} + a^2 - 2bc \right\} = 5.5716483$$

Putting the values of a , b , c , a^2 , b^2 , c^2 , R and using the matching relationship $Z_1 Z_3 = R$ and $Z_2^2 = R$, we get

$$Z_1^8 + 8.94427Z_1^7 + 28Z_1^6 + 26.832822Z_1^5 - 145.43296Z_1^4 - 80.498448Z_1^3 \dots \\ \dots - 20Z_1^2 + 44.72136Z_1 + 25 = 0 \quad \dots(2.17)$$

Comparing the coefficients of $\cos^4 \theta$

$$\frac{R}{4} \{-3c^2 - b^2 + \frac{2a(R+1)}{R} + 2a^2 - 4bc\} = 6.920826$$

Putting the values of b^2 , c^2 , bc and R the equation becomes,

$$Z_1^8 + 8.94427Z_1^7 + 24Z_1^6 + 8.944276Z_1^5 - 119.20826Z_1^4 - 62.609904Z_1^3 \dots \\ \dots + 0Z_1^2 + 44.72136Z_1 + 25 = 0 \quad \dots(2.18)$$

Comparing the coefficients of $\cos^3 \theta$

$$\frac{R}{4} \{-3c^2 + a^2 - 2bc\} = 2.149174$$

Putting the values of a^2 , c^2 , bc , and R the equation becomes

$$Z_1^8 + 8.94427Z_1^7 + 20Z_1^6 - 8.94427Z_1^5 - 92.98348Z_1^4 - 44.72136Z_1^3 + 20Z_1^2 \dots \\ \dots + 44.72136Z_1 + 25 = 0 \quad \dots(2.19)$$

We have three simultaneous equations (2.17), (2.18), (2.19).

These are the 8th order equations and they cannot be solved in an ordinary way. We want one common real root from these equations.

To do so a root finding computer subroutine is used which gives the zeros of these equations (Table 2.8).

Looking at the solution of these equations it is seen that there is one common real root which satisfies all the equations so selecting this root as value of Z_1

Table 2.8

PAGE 1

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PROGRAM ZEROS

```

PROGRAM ZEROS(INPUT,OUTPUT)
COMPLEX Z
DIMENSION A(9),Z(8)
NDEG=8
READS,(A(I),I=1,9)
CALL ZRPOLY (A,NDEG,Z,IER)
PRINT 5
FORMAT(7(/))
PRINT*,(Z(I),I=1,8)
STOP
END

```

5

10

```

(.6661845031102,0.) (-.4762649438345,0.) (-.335004135138,.6382390598971) (-.335004135138,-.6382390598971)
(1.737240528831,0.) (-2.33102885276,3.213284680255) (-2.33102885276,-3.213284680255) (-5.539366112309,0.)

```

```

(.7602271984794,0.) (-.335004088704,.6382390401813) (-.335004088704,-.6382390401813) (-.5499704753776,0.)
(1.73724067369,0.) (-2.341196743953,2.545111654865) (-2.341196743953,-2.545111654865) (-5.539365731478,0.)

```

```

(-.3350042127295,.6382390989641) (-.3350042127295,-.6382390989641) (-.6941785374349,0.) (.8903346885253,0.)
(1.73724036871,0.) (-2.334146955922,1.625333025168) (-2.334146955922,-1.625333025168) (-5.539364182496,0.)

```

$$\therefore Z_1 = 1.737240$$

$$Z_2 = \sqrt{R} = 2.2360679$$

$$Z_3 = R/Z_1 = 2.8781285$$

$$\therefore Z_1 = 1.737240$$

$$Z_2 = 2.2360679$$

$$Z_3 = 2.8781285$$

CHEBYSHEV CASE

To show the plot with these values of characteristic impedances we use the Power Loss Equation (2.14) in a computer plotting subroutine which plots the Chebyshev characteristics using the above values of characteristic impedances (Table 2.9, Fig. c2.10). This characteristic is exactly the same as compared to one plotted using equation (2.15) without using the values of characteristic impedances (Fig. c2.9)

Now the values of these characteristic impedances are changed slightly and it is seen that there is no effect of the change in the behaviour of the characteristics (Table 2.10, Fig. c2.11).

2.7 Discussion

In this chapter, Step Quarter Wave Transformer was designed for the case $n = 3$ to give Butterworth and Chebyshev response with the help of a computer. The characteristics of Butterworth and Chebyshev polynomials were discussed and using the scattering matrices the

Table 2.9

```

C      PROGRAM CHAR (INPUT,OUTPUT)
C      THIS PROGRAM PLOTS THE POWER LOSS RATIO PT FOR THE THIRD ORDER
C      CHEBYSHEV AGAINST ELECTRICAL LENGTH THETA WHICH IS NORMALIZED
C      TO TH1.
C      IN THIS PROGRAM THE CHARACTERISTIC IMPEDANCES WHICH ARE DETERMINED
C      FOR CHEBYSHEV CASE ARE USED IN POWER LOSS RATIO PT.
      DIMENSION X(161),Y(161,1),A(160),IMAG4(5151)
      PRINT 5
5      FORMAT (1H1,////,21X,*TH1*,23X,*PT*,//)
      DO 15 I=1,161
      TH=(I-1.0)*0.01
      R=5.0
      Z1=1.737240
      Z2=2.2360679
      Z3=2.8781285
      A2=Z1/Z3+Z2/Z3+Z1/Z2+Z2/(Z1*R)+Z3/(Z1*R)+Z3/(Z2*R)
      B=Z3/R+1/Z3+Z2/R+Z1/R+1/Z1+1/Z2
      C=(Z1*Z3)/(Z2*R)+Z2/(Z1*Z3)
      Q=(COS(TH))**6
      P=(COS(TH))**4
      Q=(COS(TH))**2
      PT=(R/4*(((R+1)**2/(R**2))-C**2-B**2+(2*A2*(R+1))/
1R+A2**2-2*B*C))*Q-(R/4*(-3*C**2-B**2+(2*A2*(R+1))/
1R+2*A2**2-4*B*C))*P
      S=(R/4*(-3*C**2+A2**2-2*B*C))*Q
      S=(R/4)*C**2
      TH1=TH/1.6
      X(I)=TH1
      Y(I,1)=PT
      PRINT 10,TH1,PT
10     FORMAT (15X,E15.8,10X,E15.8)
15     CONTINUE
      READ 20,(A(I),I=1,160)
20     FORMAT (80A1)
      CALL USPLH (X,Y,161,1,1,161,A,IMAG4,IER)
      STOP
      END

```

POWER LOSS RATIO PT VERSUS THETA TH1

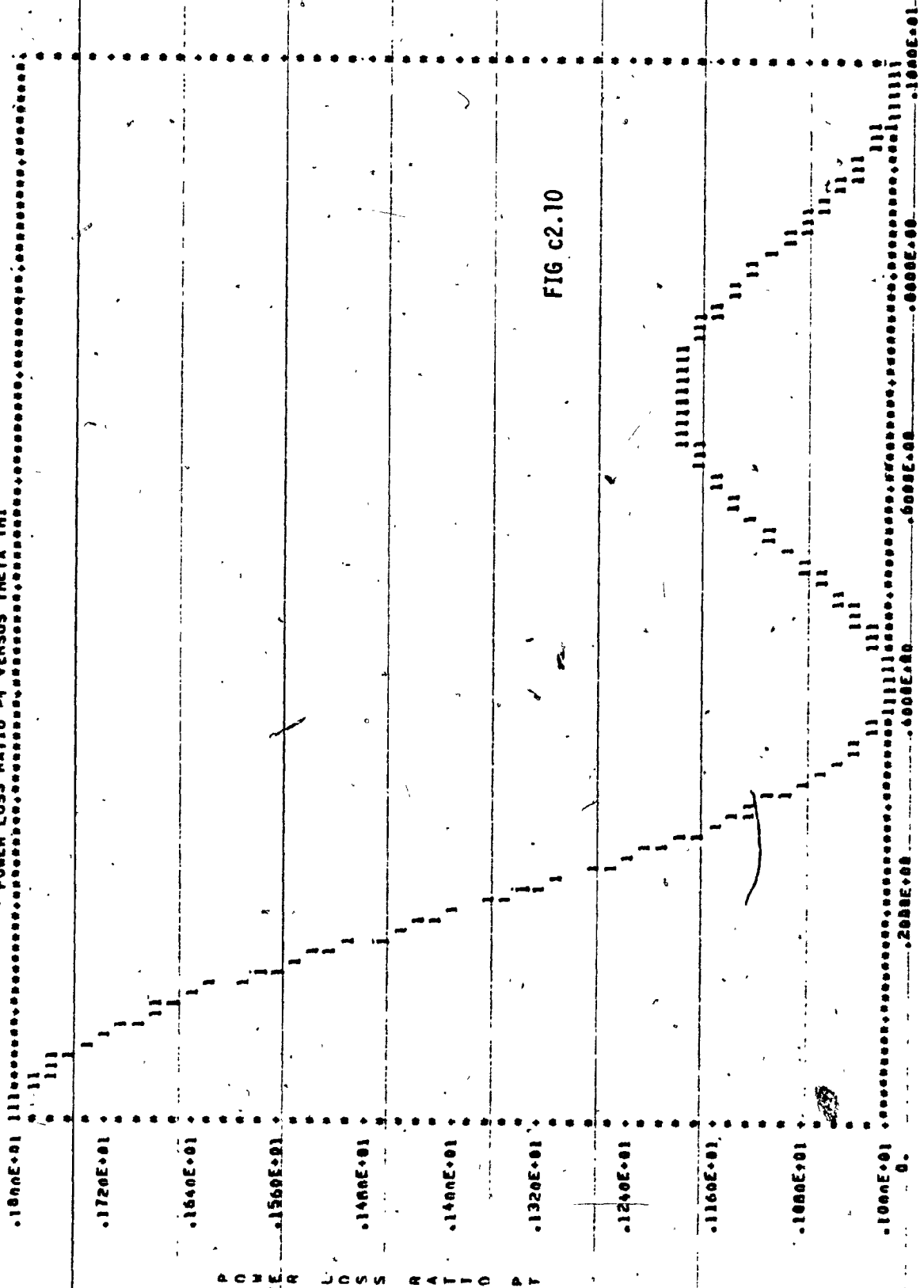


FIG C2.10

THETA NORMALIZED

Table 2.10

```

PROGRAM CHAR (INPUT,OUTPUT)
C THIS PROGRAM PLOTS THE POWER LOSS RATIO PT FOR THE THIRD ORDER
C CHEBYSHEV AGAINST ELECTRICAL LENGTH THETA WHICH IS NORMALIZED
C TO TH1.
C IN THIS PROGRAM THE CHARACTERISTIC IMPEDANCES WHICH ARE DETERMINED
C FOR CHEBYSHEV CASE ARE CHANGED SLIGHTLY AND USED IN POWER
C LOSS RATIO PT.
DIMENSION X(161),Y(161,1),A(160),IMAG4(5151)
PRINT 5
S FORMAT (1H1,////),21X,TH1,23X,PT,/)
DO 15 I=1,161
TH=(I-1.0)*0.01
R=5.0
Z1=1.837240
Z2=2.3360679
Z3=2.9781285
A2=Z1/Z3+Z2/Z3+Z1/Z2+Z2/(Z1*R)+Z3/(Z1*R)+Z3/(Z2*R)
B=Z3/R+1/Z3+Z2/R+Z1/R+1/Z1+1/Z2
C=(Z1*Z3)/(Z2*R)+Z2/(Z1*Z3)
Q=(COS(TH))**6
P=(COS(TH))**4
Q=(COS(TH))**2
PT=(R/4*((R+1)**2/(R**2))-C**2-B**2+(2*A2*(R+1))/
1R+A2**2-2*B*C))*Q-(R/4*(-3*C**2-B**2+(2*A2*(R+1))/
1R+2*A2**2-4*B*C))*P
+ (R/4*(-3*C**2+A2**2-2*B*C))*Q
+ (R/4)*C**2
TH1=TH/1.6
X(I)=TH1
Y(I,1)=PT
PRINT 10,TH1,PT
10 FORMAT (15X,E15.8,10X,E15.8)
15 CONTINUE
READ 20,(A(I),I=1,160)
20 FORMAT (80A1)
CALL USPLH (X,Y,161,1,1,161,A,IMAG4,IER)
STOP
END

```


POWER LOSS RATIO PL VERSUS THETA TH1

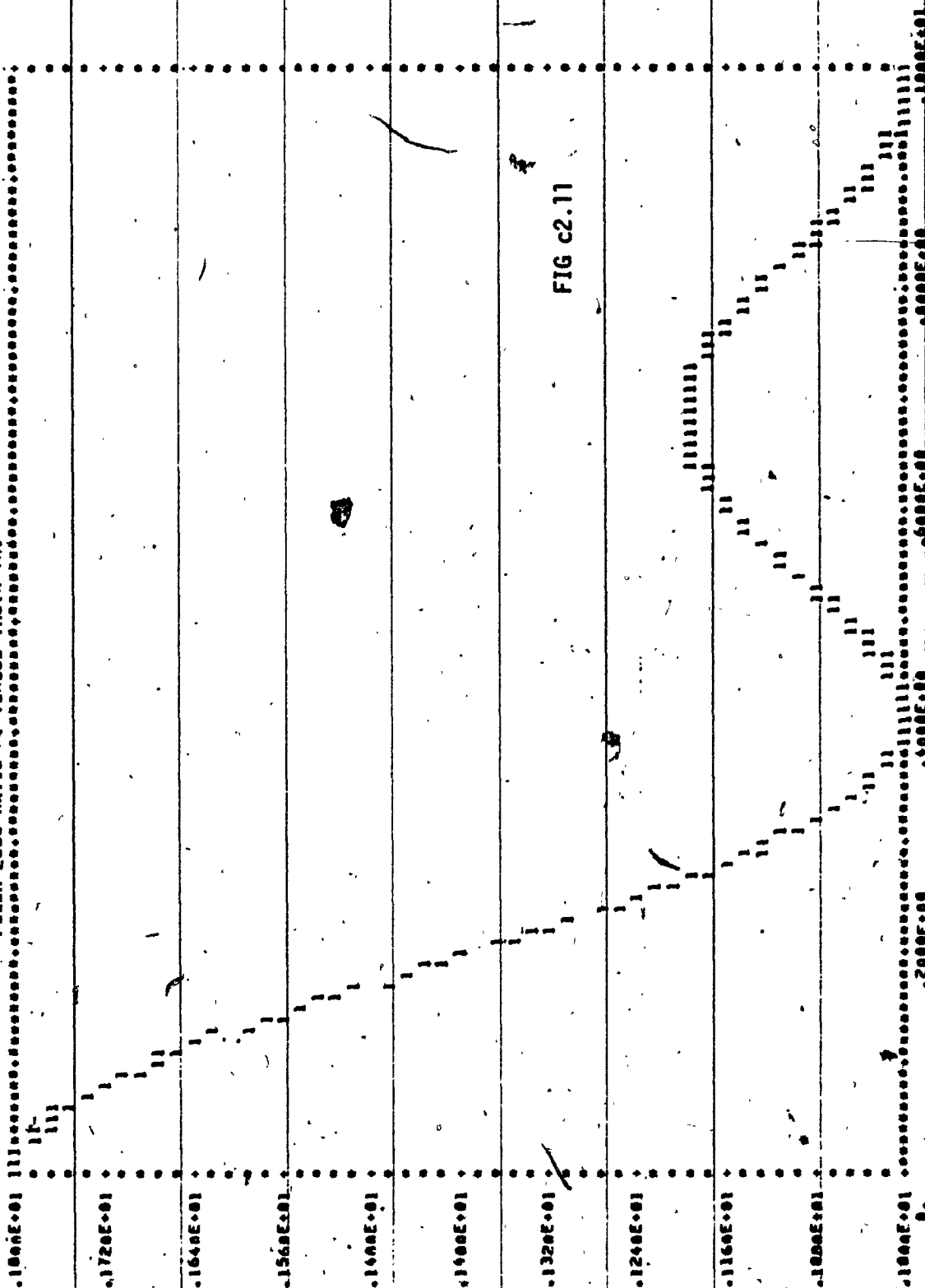


FIG c2.11

THETA NORMALIZED - - - TH1

resultant matrix was derived for n sections of transmission lines in cascade, each having the same length but different characteristic impedances.

The ratio of characteristic impedances Z_3/Z_1 in Case of Butterworth came out to be $Z_3/Z_1 = 3.33306419$, while in Chebyshev Case $Z_3/Z_1 = 1.6567247$.

CHAPTER III

ANALYSIS OF STEPPED QUARTER WAVE TRANSFORMERS WITH CAPACITORS AT JUNCTIONS

In the previous problem of Chapter II, discontinuity capacitances were not considered and their susceptance was assumed to be zero. This would be approximately true in a low frequency coaxial line, but not in a high frequency coaxial line or waveguide. The presence of discontinuity susceptances has two effects.

3.1 Effects of Discontinuity Susceptances

1. The lesser effect is a small increase in the magnitudes of the individual step reflections.
2. The greater effect is the introduction of phase angles in the reflection and transmission coefficients of the steps.

3.2 Methods of Correction for these Effects

After the addition of capacitances, the voltage reflection and transmission coefficients of the step are given, respectively, as

$$\Gamma_n = \frac{Y_n - Y_{n+1} - jB_m}{Y_n + Y_{n+1} + jB_m}$$

or

$$\Gamma_n = \frac{Y_n/Y_{n+1} - 1 - jB_m/Y_{n+1}}{Y_n/Y_{n+1} + 1 + jB_m/Y_{n+1}}$$

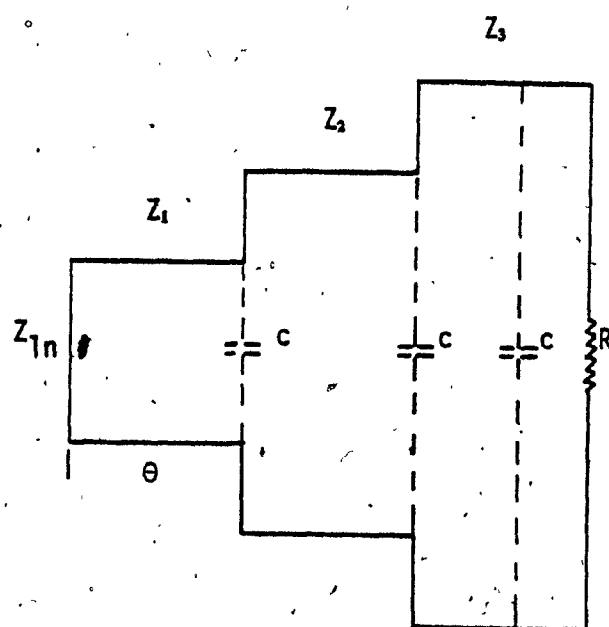


FIG 3.1

STEPPED QUARTER WAVE TRANSFORMER
WITH CAPACITANCES AT JUNCTIONS

and
$$T_n = \frac{2Y_n}{Y_n + Y_{n+1} + jB_m}$$

or
$$T_n = \frac{2Y_n/Y_{n+1}}{Y_n/Y_{n+1} + 1 + jB_m/Y_{n+1}}$$

Circuit of a Single Step is as shown in Fig. 3.2.

The magnitudes of Γ_n and T_n are as follows:

$$|\Gamma_n| = \sqrt{\frac{(Y_n/Y_{n+1} - 1)^2 + (B_m/Y_{n+1})^2}{(Y_n/Y_{n+1} + 1)^2 + (B_m/Y_{n+1})^2}}$$

and
$$|T_n| = \sqrt{\frac{(2Y_n/Y_{n+1})^2}{(Y_n/Y_{n+1} + 1)^2 + (B_m/Y_{n+1})^2}}$$

It can be seen that Γ_n is not very much effected by B_m , if $(B_m/Y_{n+1})^2 \ll (Y_n/Y_{n+1} - 1)^2$. This would be in the case of low frequency coaxial line.

The effect of B_m on T_n is very less and can be neglected in all cases.

The Phase angles of Γ_n and T_n are as follows:

$$\angle \Gamma_n = -\tan^{-1} \left[B_m/Y_{n+1} / (Y_n/Y_{n+1} - 1) \right] - \tan^{-1} \left[B_m/Y_{n+1} / (Y_n/Y_{n+1} + 1) \right]$$

$$\angle T_n = -\tan^{-1} \left[B_m/Y_{n+1} / (Y_n/Y_{n+1} + 1) \right]$$

Considering the individual waves reflected from the steps arriving at some particular reference point between the generator

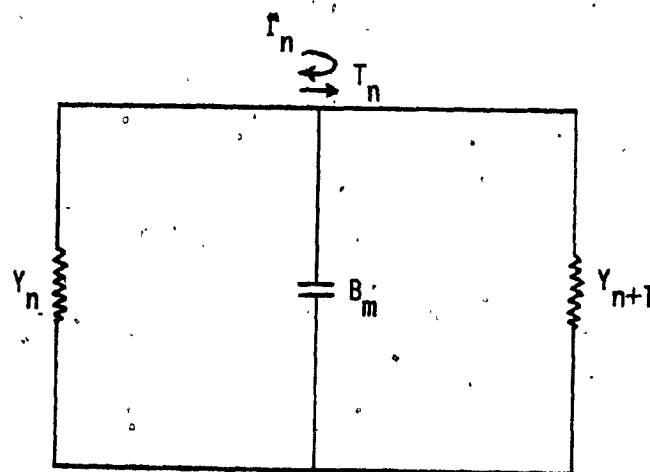


FIG 3.2
EQUIVALENT CIRCUIT OF A SINGLE STEP

and the transformer. Now the phase angles of the reflection and transmission coefficients cause phase shifts in these returning waves in addition to those due to the distance travelled.

These extra phase shifts are given by

$$\phi_n = \angle \Gamma_n - 2\angle T_1 - 2\angle T_2 \dots - 2\angle T_{n-1}$$

These extra phase shifts may be eliminated at the center of the band where $\phi = 90^\circ$ by moving each step towards the generator by an electrical length equal to one-half of the extra phase shift.

The distance by which each step is moved is therefore,

$$x_n = \frac{\phi_n}{2\beta}$$

where

$$\beta = \frac{360}{\lambda} \text{ degrees}$$

Now we have

$$\frac{\lambda}{4} = \frac{\pi}{2} \text{ degrees}$$

$$\therefore \lambda = 2\pi$$

$$\therefore \beta = \frac{2\pi}{2\pi} = 1$$

$$\therefore x_n = \frac{\phi_n}{2}$$

Although this correction is made only at center of the band, it should give good results over a wide range in ϕ . The physical result of this correction is in most cases to decrease the spacings between steps to somewhat less than $\lambda/4$.

3.3 BUTTERWORTH CASE

Without the shunt capacitance, the characteristic impedances in case of Butterworth are

$$Z_1 = 1.225239656$$

$$Z_2 = 2.2360679$$

$$Z_3 = 4.0808344$$

$$\text{Terminating impedance } R = 5$$

$$\therefore Y_1 = 0.8161668$$

$$Y_2 = 0.4472136$$

$$Y_3 = 0.2450479$$

$$Y_4 = 0.2 \quad \{1/R\}$$

The equation in this case for Power Loss Ratio is

$$P_T = 1 + AK \cos^{2n} \theta$$

$$\therefore P_T = 1 + AK \cos^4 \theta$$

where

$$AK = \frac{(R - 1)^2}{4R} = 0.8$$

$$\text{and } \theta = \theta - \phi_3 / 2$$

In this case to observe the effect of shunt capacitance a computer program (Table 3.1, 3.2, 3.3) is written which test the effect of Shunt capacitance for different frequency. Three values of capacitance are tested in this computer program which calculates new values of Transmission and Reflection coefficients, new values of Z_1 , Z_2 , and Z_3 which will be changed by discontinuity capacitance. This program also shows the extra phase shift introduced by ϕ_3 and

the distance moved x_s to compensate for this extra phase shift and finally the characteristics are plotted, (Figs. c3.1, c3.2, c3.3, c3.4, c3.5, c3.6, c3.7, c3.8, c3.9, c3.10, c3.11, c3.12).

3.4 Chebyshev Case

Now coming to our previous problem without the shunt capacitances, we had the characteristic impedances for Chebyshev Case as

$$Z_1 = 1.737240$$

$$Z_2 = 2.2360679$$

$$Z_3 = 2.8781285$$

$$\text{Terminating impedance } R = 5$$

Therefore

$$Y_1 = 0.5756257$$

$$Y_2 = 0.4472136$$

$$Y_3 = 0.347448$$

$$Y_4 = 0.2 \quad \{1/R\}$$

To observe the effect of Shunt capacitance, three different capacitance values are taken and they are tested for different frequency values. This is all done by a computer program (Table 3.4, 3.5, 3.6) which calculates the susceptance B_m , the new value of Transmission and Reflection coefficient, it also shows the extra phase shift introduced due to the Shunt capacitance and the distance moved to correct this extra phase shift. It also plots the Characteristic

of Chebyshev case using plotting subroutine (Figs. c3.13, c3.14, c3.15, c3.16, c3.17, c3.18, c3.19, c3.20, c3.21, c3.22, c3.23, c3.24).

Now our original θ becomes as

$$\theta' = \theta - x_3 \quad \text{as } x_3 = \phi_3/2$$

$$\therefore \theta' = \theta - \phi_3/2$$

$$\therefore P_T = 1 + AK T_n^2 [\cos \theta' / P]$$

This computer program also gives the new values of characteristic impedances Z_1 , Z_2 and Z_3 which will be changed slightly depending on shunt capacitance and frequency. The new values are calculated from new reflected coefficient Γ_n .

3.5 Discussion

In this Chapter, the effect of capacitances at junctions due to discontinuity were discussed and methods were applied for correction of these effects and finding the new values of characteristic impedances with the help of a computer. Three different capacitance values were taken and they were tested for different frequencies for both Butterworth and Chebyshev Cases with the aid of a computer program. It was found that as the frequency increases the effect of capacitance due to discontinuity increases on the values of Transmission and Reflection coefficients, characteristic impedances and on the phase shift. It was shown that the change in the characteristic impedances due to these capacitance effects was not high.

Table 3.1

```

PROGRAM EFFECT (INPUT,OUTPUT)
THIS PROGRAM OBSERVES THE EFFECT OF SHUNT
CAPACITANCE ON TRANSMISSION AND REFLECTION
COEFFICIENTS.
IT CALCULATES THE NEW VALUES OF TRANSMISSION
AND REFLECTION COEFFICIENTS.
IT CALCULATES THE EXTRA PHASE SHIFTS INTRODUCED
DUE TO CAPACITANCE.
IT PLOTS THE POWER LOSS RATIO PT FOR THE THIRD
ORDER BUTTERWORTH AGAINST ELECTRICAL LENGTH
THETA WHICH IS NORMALIZED TO TH1 WHERE
TH1=THA/THN, AND THETA IS A FUNCTION OF FREQ.
THE PASSBAND TOLERANCE IN CASE OF BUTTERWORTH
IS AK=0.8.
B IS THE SUSCEPTANCE OF THE CAPACITOR.
Y4=1/R, WHERE R IS THE TERMINATING RESISTANCE, R=5.
IT GIVES THE NEW VALUES OF CHARACTERISTIC
IMPEDANCES EFFECTED BY DISCONTINUITY CAPACITANCE.
DIMENSION X(161),Y(161,1),A(160),IMAG4(5151)
Y1=0.8161668
Y2=0.4472136
Y3=0.2450479
Y4=0.2
AK=0.8
C=10.0E-12
READ*,W
B=W*C
PRINT 5,W,C,B
5  FORMAT(1H1,/////,10X,*W=*,E15.8,5X,*C=*,E15.8,5X,
    *B=*,E15.8)
    T1=SQRT(((2.0*Y1)/Y2)**2/((Y1/Y2+1)**2+(B/Y2)**2))
    R1=SQRT(((Y1/Y2-1)**2+(B/Y2)**2)/((Y1/Y2+1)**2
    +(B/Y2)**2))
    PRINT 10,T1,R1
10  FORMAT(///,10X,*T1=*,E15.8,5X,*R1=*,E15.8)
    YN=((1+R1)*Y2)/(1-R1)
    Z1=1/YN
    Z2=(5.0)**0.5
    Z3=5.0/Z1
    PRINT 12,Z1,Z2,Z3
12  FORMAT(///,10X,*NEW Z1=*,E15.8,5X,*NEW Z2=*,
    *E15.8,5X,*NEW Z3=*,E15.8)
    AT1=-ATAN((B/Y2)/((Y1/Y2+1)))
    AT2=-ATAN((B/Y3)/((Y2/Y3+1)))
    AR3=-ATAN((B/Y4)/((Y3/Y4+1)))
    PH13=(-AR3-AT1-2*AT2)
    X3=PH13/2
    PRINT 15,X3
15  FORMAT(///,10X,*EXTRA PHASE SHIFT PH13=*,E15.8,
    *5X,*DISTANCE MOVED X3=*,E15.8,///)

```

```
      THN=(1.6-X3)
      PRINT 20
20    FORMAT(16X,*TH1*,22X,*PT*,//)
      DO 30 I=1,161
      TH=(I-1.0)*0.01
      THA=ABS(TH-X3)
      Q=(COS(THA))**4
      PT=1.0+AK*Q
      TH1=THA/THN
      X(I)=TH1
      Y(I,1)=PT
      PRINT 25,TH1,PT
25    FORMAT(10X,E15.8,10X,E15.8)
30    CONTINUE
      READ 40,(A(I),I=1,160)
40    FORMAT(80A1)
      CALL USPLH (X,Y,161,1,1,161,A,IMAG4,IER)
      STOP
      END
```

W= .10000000E+09 C= .10000000E-10 B= .10000000E-02

Y1= .12920384E+01 R1= .29203749E+00

NEW Z1= .12252371E+01 NEW Z2= .22360680E+01 NEW Z3= .40808428E+01

EXTRA PHASE SHIFT PH13= .28914027E-01 DISTANCE MOVED X3= .14457013E-01

THI

PT

.91180205E-02	.17994985E+01
.28110329E-02	.17999523E+01
.34959547E-02	.17999263E+01
.98029424E-02	.17994204E+01
.18109930E-01	.17984355E+01
.22416918E-01	.17969732E+01
.28723905E-01	.17958358E+01
.35030893E-01	.17926263E+01
.41337881E-01	.17897487E+01
.47644868E-01	.17864076E+01
.53951856E-01	.17826081E+01
.60258843E-01	.17783564E+01
.66565831E-01	.17736593E+01
.72872819E-01	.17685240E+01
.79179806E-01	.17629587E+01
.85486794E-01	.17569721E+01
.91793782E-01	.17505737E+01
.98100769E-01	.17437733E+01
.10440775E+00	.17365836E+01
.11071474E+00	.17290097E+01
.11702173E+00	.17210692E+01
.12332872E+00	.17127723E+01
.12963571E+00	.17041316E+01
.13594269E+00	.16951604E+01
.14224968E+00	.16858721E+01
.14855667E+00	.16762806E+01
.15486366E+00	.16664002E+01
.16117065E+00	.16562455E+01

POWER LOSS RATIO PJ VERSUS THETA TH1

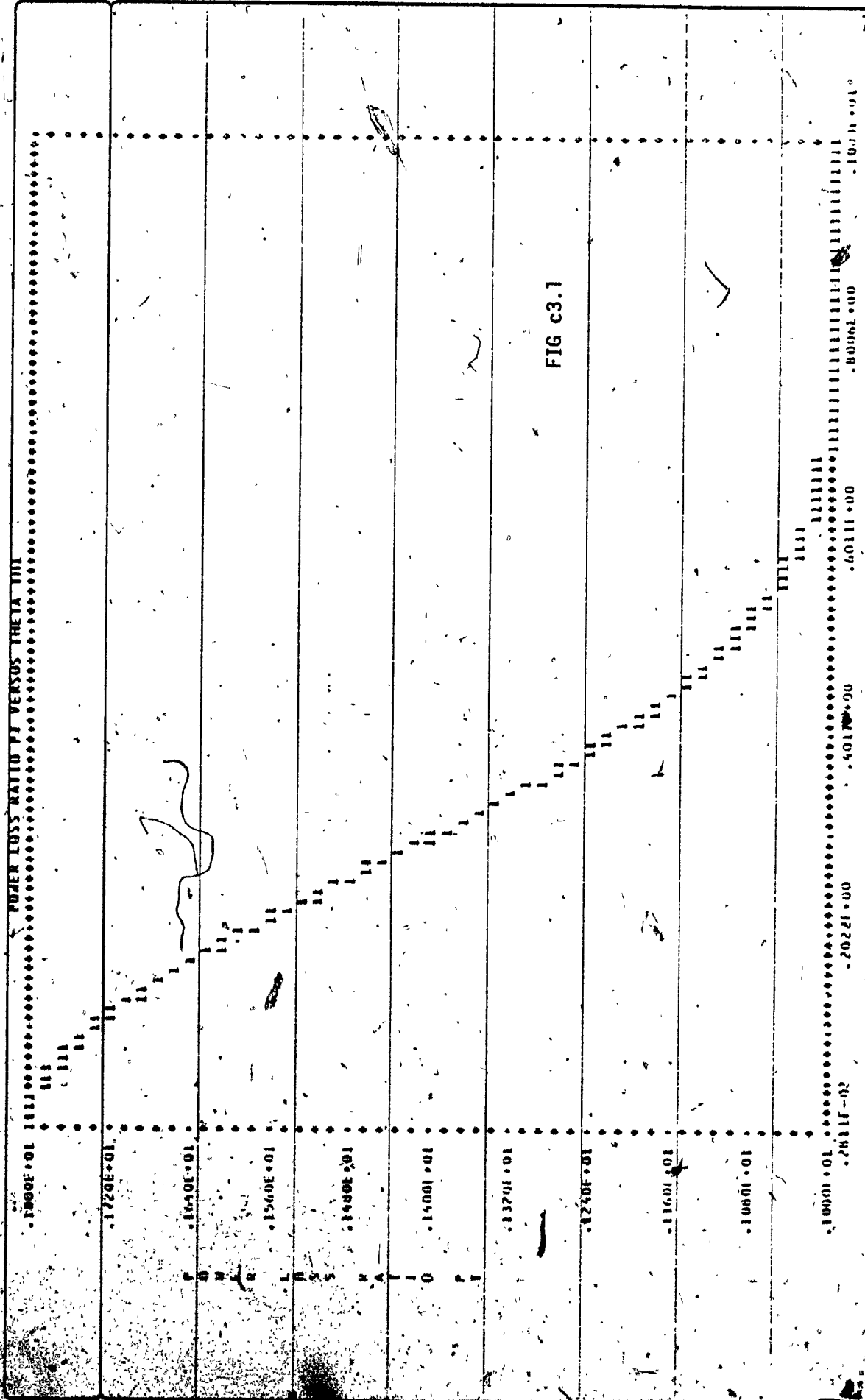


FIG C3.1

W = .20000000E+09 C = .10000000E+1J S = .20000000E+02

T1 = .12920442E+01 R1 = .129204043E+00

NEW Z1 = .1292292E+01 NEW Z2 = .22350680E+01 NEW Z3 = .40808490E+01

EXTRA PHASE SHIFT PH13 = .57806172E-01 DISTANCE MOVED A3 = .24903085E-01

THI

PT

.18398755E-01	.17979973E+01
.12031776E-01	.17991428E+01
.56667960E-02	.17998098E+01
.69818367E-03	.17999971E+01
.70631834E-02	.17997045E+01
.13428143E-01	.17989324E+01
.19793123E-01	.17976821E+01
.26158103E-01	.17959556E+01
.32521082E-01	.17937556E+01
.35888062E-01	.17910857E+01
.45253042E-01	.17879300E+01
.51618021E-01	.17843316E+01
.57983001E-01	.17803022E+01
.64347981E-01	.17758022E+01
.70712961E-01	.17708607E+01
.77077940E-01	.17654856E+01
.83442920E-01	.17596852E+01
.89807900E-01	.17534687E+01
.96172840E-01	.17468458E+01
.10253786E+00	.17398267E+01
.10890284E+00	.17324224E+01
.11526752E+00	.17246443E+01
.12153283E+00	.17165043E+01
.12799775E+00	.17080149E+01
.13436278E+00	.16991391E+01
.14072774E+00	.16900400E+01
.14709272E+00	.16805815E+01
.15345770E+00	.16708274E+01

POWER LOSS RATIO PL VERSUS THETA THL

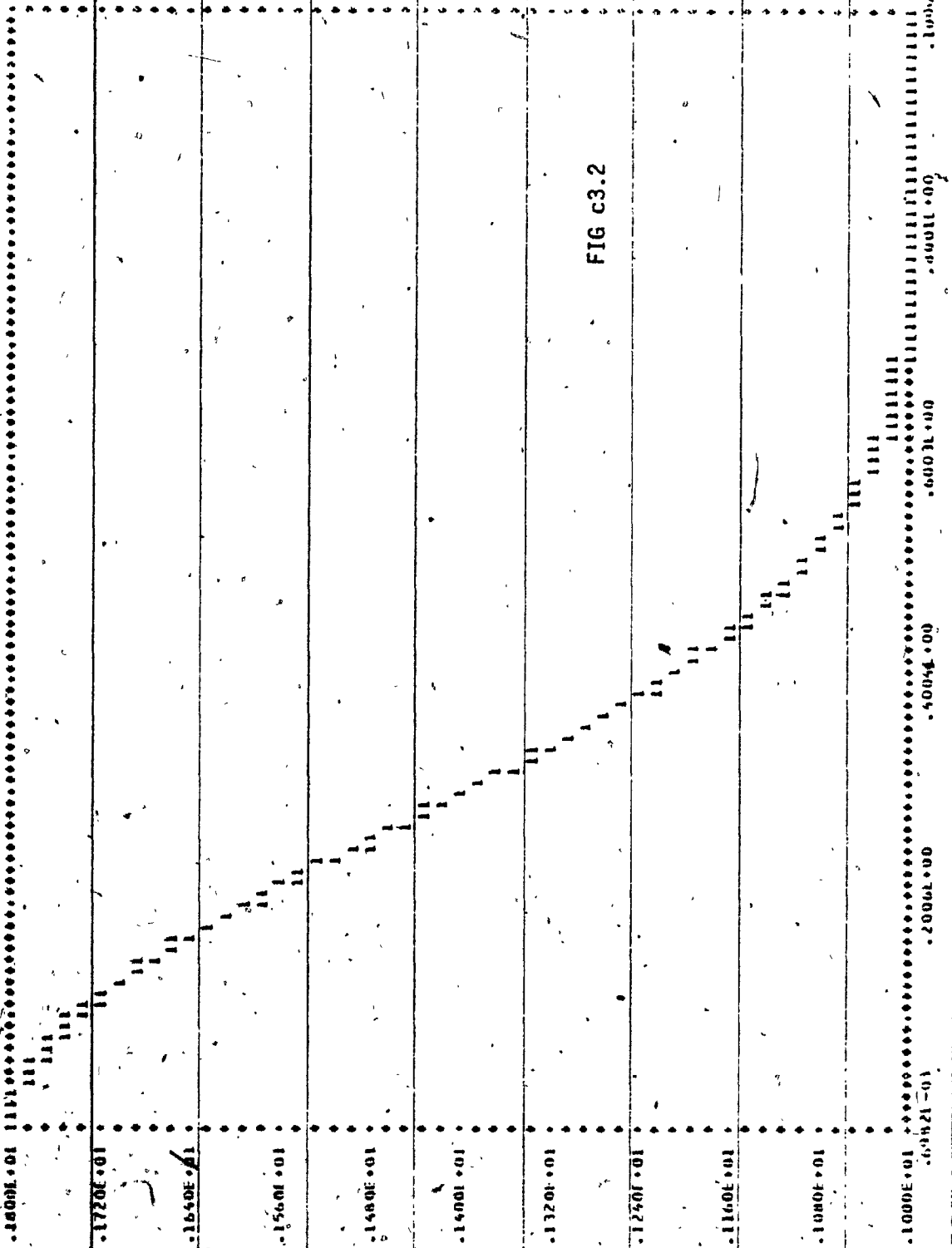


FIG C3.2

X = .40000000E-09 C = .10000000E-10 S = .40000000E-02

T1 = .12920675E+01 R1 = .29205221E+00

NEW Z1 = .12251977E+01 NEW Z2 = .22360680E+01 NEW Z3 = .40309741E+01

EXTRA PHASE SHIFT PH13 = .11543805E+00 DISTANCE MOVED X3 = .57719027E+01

TH1	PT
.37424457E-01	.17920399E+01
.30940554E-01	.17945515E+01
.24456651E-01	.17965919E+01
.17972743E-01	.17981579E+01
.11488849E-01	.17992468E+01
.50049424E-02	.17998570E+01
.14789603E-02	.17999875E+01
.79628634E-02	.17996381E+01
.14446766E-01	.17988033E+01
.20930669E-01	.17975025E+01
.27414372E-01	.17957198E+01
.33894475E-01	.17934639E+01
.40382373E-01	.17907386E+01
.46866290E-01	.17875481E+01
.53390143E-01	.17838975E+01
.59934086E-01	.17797927E+01
.66317949E-01	.17752408E+01
.72801992E-01	.17702468E+01
.79285793E-01	.17648204E+01
.85769643E-01	.17589707E+01
.92253600E-01	.17527055E+01
.98737503E-01	.17460351E+01
.10522141E+00	.17389699E+01
.11170531E+00	.17315207E+01
.11813921E+00	.17236991E+01
.12467311E+00	.17155917E+01
.13115702E+00	.17069872E+01
.13764092E+00	.16981224E+01

POWER LOSS RATIO PT VERSUS THETA TH1

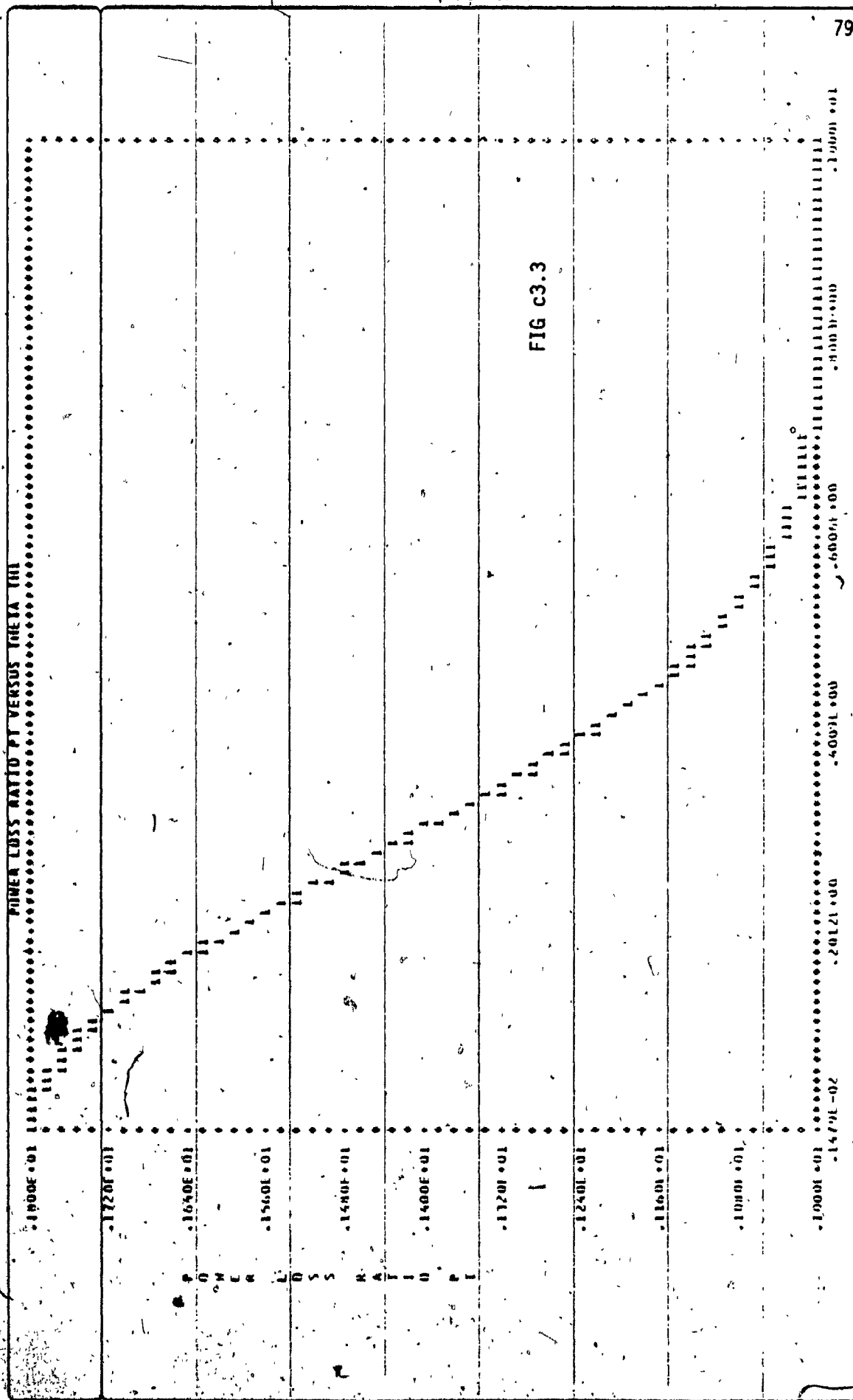


FIG C3.3

THETA NORMALIZED

M = .50009000E+03 C = .10000000E-10 S = .50004000E-22

T1 = .12920449E+01 A1 = .29206104E+00

NEW Z1 = .12291740E+01 NEW Z2 = .22360640E+01 NEW Z3 = .40810529E+01

EXTRA PHASE SHIFT PH13 = .14413519E+00 DISTANCE MOVED 43 = .72067534E-01

TH1

PT

.47166733E-01	.17876210E+01
.40521946E-01	.17908016E+01
.34077154E-01	.17935170E+01
.27532362E-01	.17957628E+01
.20987570E-01	.17975354E+01
.14442778E-01	.17983320E+01
.75979894E-02	.17996506E+01
.13531933E-02	.17999897E+01
.91915943E-02	.17998490E+01
.11736391E-01	.17992286E+01
.19291183E-01	.17981294E+01
.24325975E-01	.17965933E+01
.31170767E-01	.17945024E+01
.37915554E-01	.17919812E+01
.44460351E-01	.17889924E+01
.51005144E-01	.17855412E+01
.57549936E-01	.17816331E+01
.64094729E-01	.17772743E+01
.70639525E-01	.17724718E+01
.77194312E-01	.17672330E+01
.83729104E-01	.17615662E+01
.90273896E-01	.17554803E+01
.96818688E-01	.17489849E+01
.10336344E+00	.17420901E+01
.10990427E+00	.17348063E+01
.11645304E+00	.17271454E+01
.12299786E+00	.17191186E+01
.12954259E+00	.17107344E+01

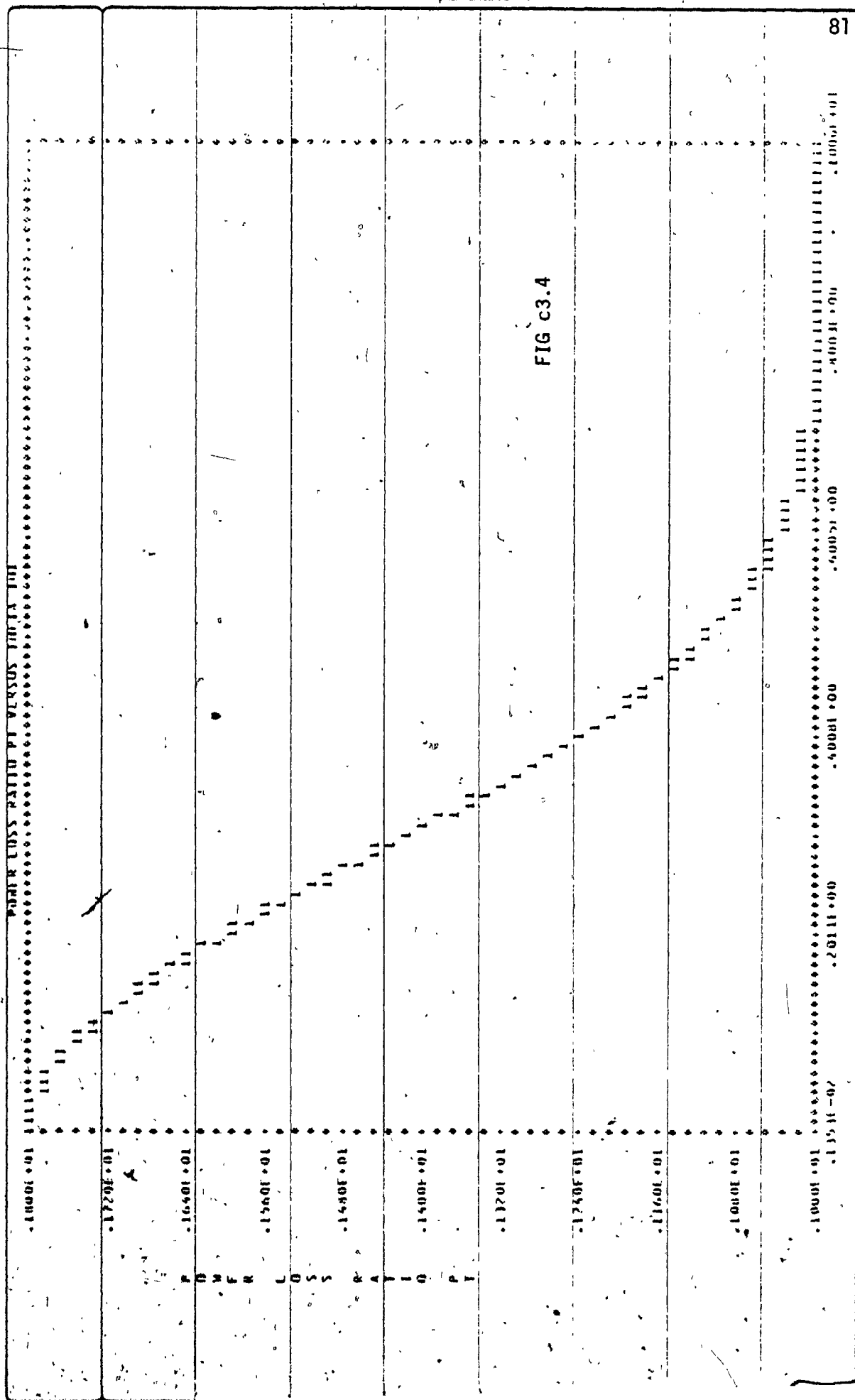


FIG C3.4

Table 3:2

```

PROGRAM EFFECT (INPUT,OUTPUT).
C   THIS PROGRAM OBSERVES THE EFFECT OF SHUNT
C   CAPACITANCE ON TRANSMISSION AND REFLECTION
C   COEFFICIENTS.
C   IT CALCULATES THE NEW VALUES OF TRANSMISSION
C   AND REFLECTION COEFFICIENTS.
C   IT CALCULATES THE EXTRA PHASE SHIFTS INTRODUCED
C   DUE TO CAPACITANCE.
C   IT PLOTS THE POWER LOSS RATIO PT FOR THE THIRD
C   ORDER BUTTERWORTH AGAINST ELECTRICAL LENGTH
C   THETA WHICH IS NORMALIZED TO TH1 WHERE
C   TH1=THA/THN, AND THETA IS A FUNCTION OF FREQ.
C   THE PASSBAND TOLERANCE IN CASE OF BUTTERWORTH
C   IS AK=0.8.
C   B IS THE SUSCEPTANCE OF THE CAPACITOR.
C   Y4=1/R, WHERE R IS THE TERMINATING RESISTANCE, R=5.
C   IT GIVES THE NEW VALUES OF CHARACTERISTIC
C   IMPEDANCES EFFECTED BY DISCONTINUITY CAPACITANCE.
C   DIMENSION X(161),Y(161,1),A(160),IMAG4(5151)
Y1=0.8161668
Y2=0.4472136
Y3=0.2450479
Y4=0.2
AK=0.8
C=15.0E-12
READ*,W
B=W*C
PRINT 5,W,C,B
5  FORMAT(1H1,/////,10X,*W=*,E15.8,5X,*C=*,E15.8,5X,
  *B=*,E15.8)
  T1=SQRT(((2.0*Y1)/Y2)**2/(Y1/Y2+1)**2+(B/Y2)**2)
  R1=SQRT(((Y1/Y2-1)**2+(B/Y2)**2)/((Y1/Y2+1)**2
  +(B/Y2)**2))
  PRINT 10,T1,R1
10  FORMAT(///,10X,*T1=*,E15.8,5X,*R1=*,E15.8)
  YN=((1+R1)*Y2)/(1-R1)
  Z1=1/YN
  Z2=(5.0)**0.5
  Z3=5.0/Z1
  PRINT 12,Z1,Z2,Z3
12  FORMAT(///,10X,*NEW Z1=*,E15.8,5X,*NEW Z2=*,
  *E15.8,5X,*NEW Z3=*,E15.8)
  AT1=-ATAN((B/Y2)/(Y1/Y2+1))
  AT2=-ATAN((B/Y3)/(Y2/Y3+1))
  AR3=-ATAN((B/Y4)/(Y3/Y4-1))
  *- ATAN((B/Y4)/(Y3/Y4+1))
  PHI3=(-AR3 -2*AT1 -2*AT2)
  X3=PHI3/2.
  PRINT 15,PHI3,X3
15  FORMAT(///,10X,*EXTRA PHASE SHIFT PHI3=*,E15.8,
  *5X,*DISTANCE MOVED X3=*,E15.8,///)

```

```
THN=(1.6-X3)
PRINT 20
20  FORMAT(16X,*TH1*,22X,*PT*,//)
    DO 30, I=1,161
      TH=(I-1.0)*0.01
      THA=ABS(TH-X3)
      Q=(COS(THA))**6
      PT=1.0+AK*Q
      TH1=THA/THN
      X(I)=TH1
      Y(I,1)=PT
      PRINT 25,TH1,PT
25  FORMAT(10X,E15.8,10X,E15.8)
30  CONTINUE
    READ 40,(A(I),I=1,160)
40  FORMAT (80A1)
    CALL USPLH (X,Y,161,1,1,161,A,IMAG4,IER)
    STOP
    END
```

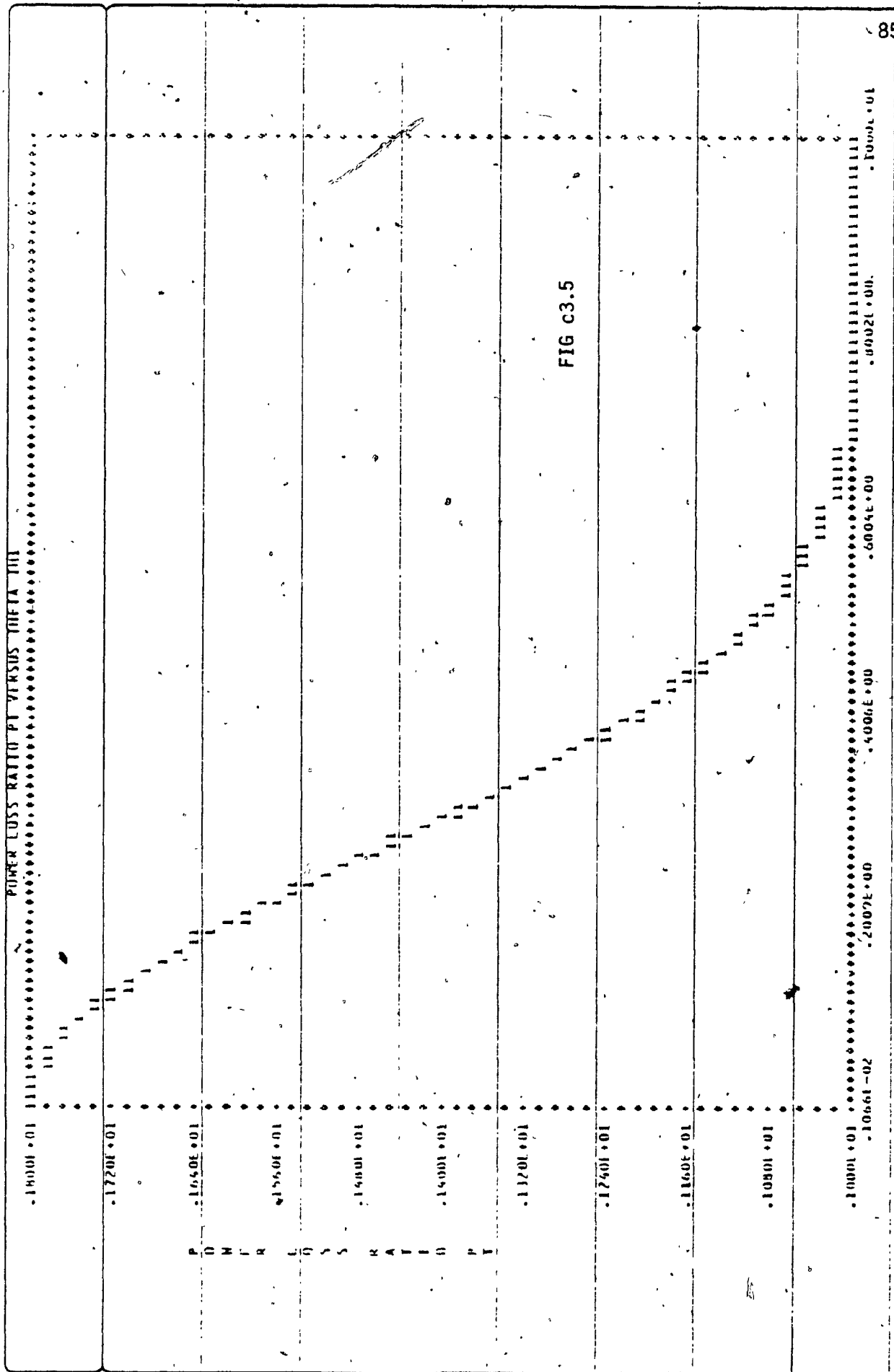
W = .10000000E+00 C = .10000000E-10 3 = .15000000E-02

T1 = .12920409E+01 Z1 = .29203872E+00

NEW Z1 = .12252338E+01 NEW Z2 = .22160560E+01 NEW Z3 = .40308537E+01

EXTRA PHASE SHIFT PH13 = .4336+199E-01 DISTANCE MOVED X3 = .215+2779E-01

THL	PT
.13737473E-01	.17988724E+01
.74016137E-02	.17996725E+01
.10657545E-02	.17999932E+01
.52701047E-02	.17998340E+01
.11605944E-01	.17991951E+01
.17941323E-01	.17980775E+01
.24277542E-01	.17984811E+01
.30613542E-01	.17944143E+01
.36949401E-01	.17918746E+01
.43285260E-01	.17888673E+01
.49621119E-01	.17853989E+01
.55956971E-01	.17814734E+01
.62242935E-01	.17770974E+01
.68628697E-01	.17722773E+01
.74964556E-01	.17676224E+01
.81380415E-01	.17613393E+01
.87636274E-01	.17552375E+01
.93972114E-01	.17487265E+01
.10030743E+00	.17418164E+01
.10664395E+00	.17345131E+01
.11297971E+00	.17268427E+01
.11931597E+00	.17189020E+01
.12565143E+00	.17104385E+01
.13193729E+00	.17016748E+01
.13822316E+00	.16926142E+01
.14465901E+00	.16832403E+01
.15099487E+00	.16735672E+01
.15733073E+00	.16636092E+01



W = .20000000E+09 C = .15000000E-10 B = .30000000E-02

TL = .12920539E+01 RL = .29204534E+00

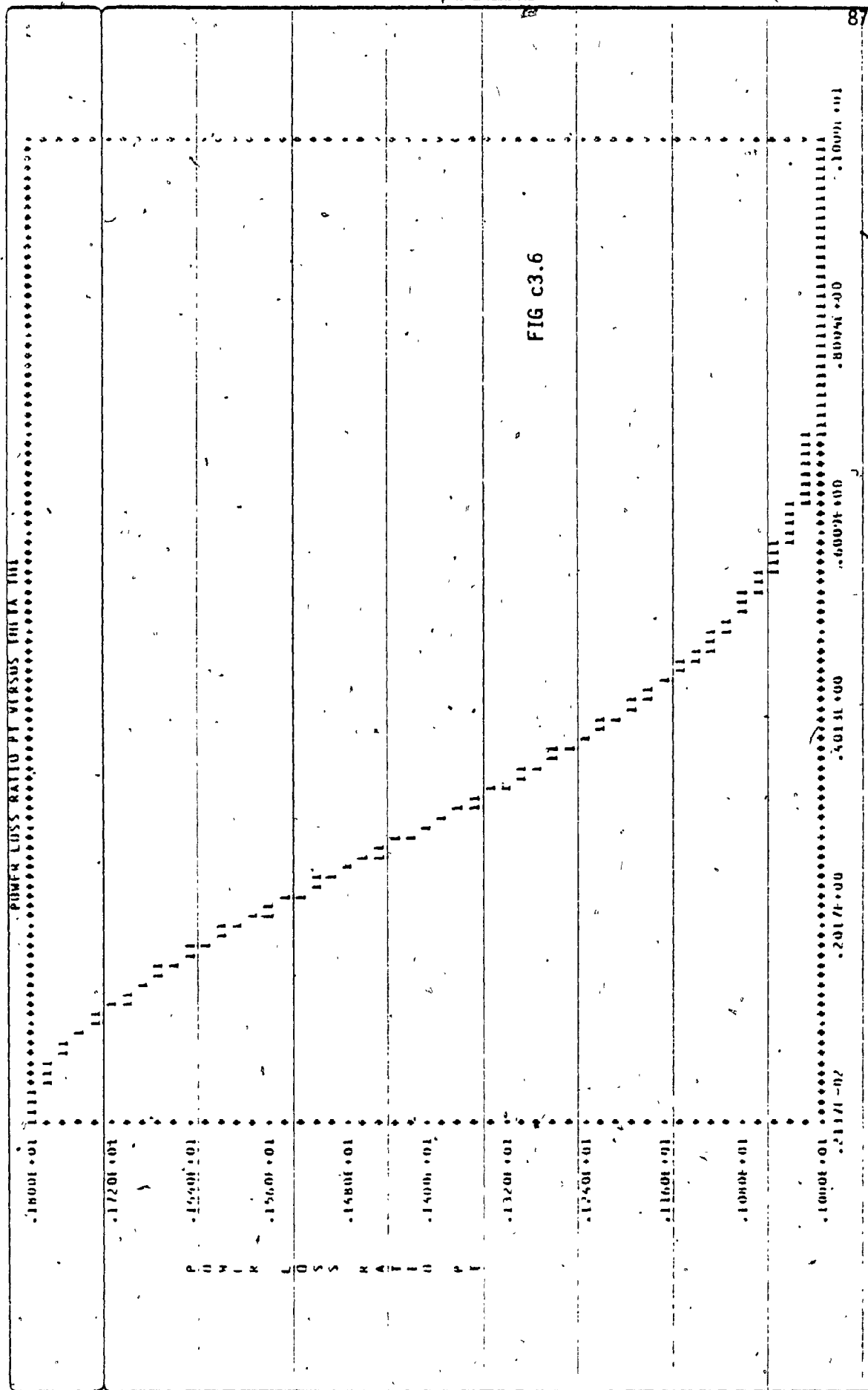
NEW Z1 = .12252161E+01 NEW Z2 = .22360680E+01 NEW Z3 = .40309123E+01

EXTRA PHASE SHIFT PH13 = .26654680E-01 DISTANCE MOVED X3 = .43327340E-01

THL

PT

.27333302E-01	.17955058E+01
.21409344E-01	.17973382E+01
.14985386E-01	.17986950E+01
.85614275E-02	.17995738E+01
.21174693E-02	.17999734E+01
.42864899E-02	.17999931E+01
.10715447E-01	.17993331E+01
.17134405E-01	.17982942E+01
.23558363E-01	.17967731E+01
.29982321E-01	.17947971E+01
.36406279E-01	.17923246E+01
.42830239E-01	.17893944E+01
.49234176E-01	.17860012E+01
.55673154E-01	.17821504E+01
.62102112E-01	.17778460E+01
.68526070E-01	.17731009E+01
.74950023E-01	.17679167E+01
.81373985E-01	.17623033E+01
.87797945E-01	.17562598E+01
.94221903E-01	.17493254E+01
.10064565E+00	.17429803E+01
.10706982E+00	.17357451E+01
.11349379E+00	.17281309E+01
.11991774E+00	.17201495E+01
.12634167E+00	.17115131E+01
.13275965E+00	.17031345E+01
.13918461E+00	.16941263E+01
.14561337E+00	.16848035E+01



THETA NORMALIZED - - - - -

W = .60000000E+09 C = .15000000E-17 B = .60000000E-02

T1 = .12921062E+01 R1 = .29207183E+00

NEW Z1 = .12251451E+01 NEW Z2 = .22360580E+01 NEW Z3 = .40811472E+01

EXTRA PHASE SHIFT PH13 = .17272542E+00 DISTANCE MOVED X3 = .86362710E-01

TH1

PT

.57056410E-01	.17322755E+01
.50449804E-01	.17861133E+01
.43843205E-01	.17894922E+01
.37236603E-01	.17924080E+01
.30630000E-01	.17948560E+01
.24023393E-01	.17968322E+01
.17416795E-01	.17983330E+01
.10810192E-01	.17993577E+01
.42035893E-02	.17999028E+01
.24030127E-02	.17999682E+01
.70096153E-02	.17999538E+01
.15616218E-01	.17986601E+01
.22222320E-01	.17972866E+01
.23829423E-01	.17954415E+01
.35436025E-01	.17931217E+01
.42042623E-01	.17903330E+01
.48649211E-01	.17870796E+01
.55255333E-01	.17833673E+01
.51862435E-01	.17792013E+01
.68469033E-01	.17745884E+01
.75075641E-01	.17695360E+01
.81682244E-01	.17640520E+01
.88288346E-01	.17581450E+01
.94895449E-01	.17513243E+01
.10130203E+00	.17430793E+01
.10810465E+00	.17379813E+01
.11471525E+00	.17304814E+01
.12132135E+00	.17226132E+01

POWER LOSS RATIO PL VERSUS FREQUENCY

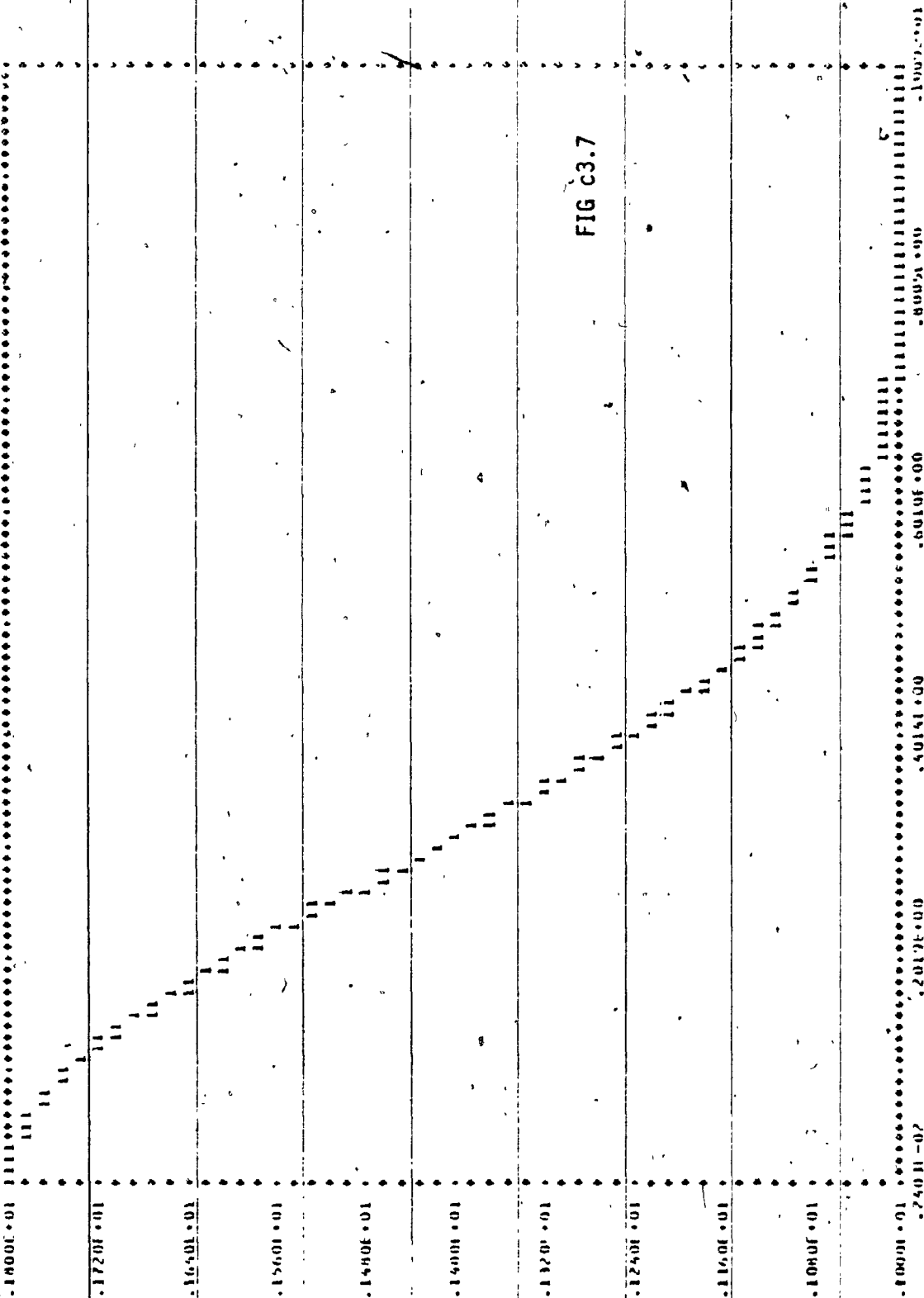


FIG C3.7

PL VERSUS FREQUENCY

* = .50000000E+09 -C = .15000000E-10 z = .75000000E-12

T1 = .12921453E+01 <1 = .24209159E+00

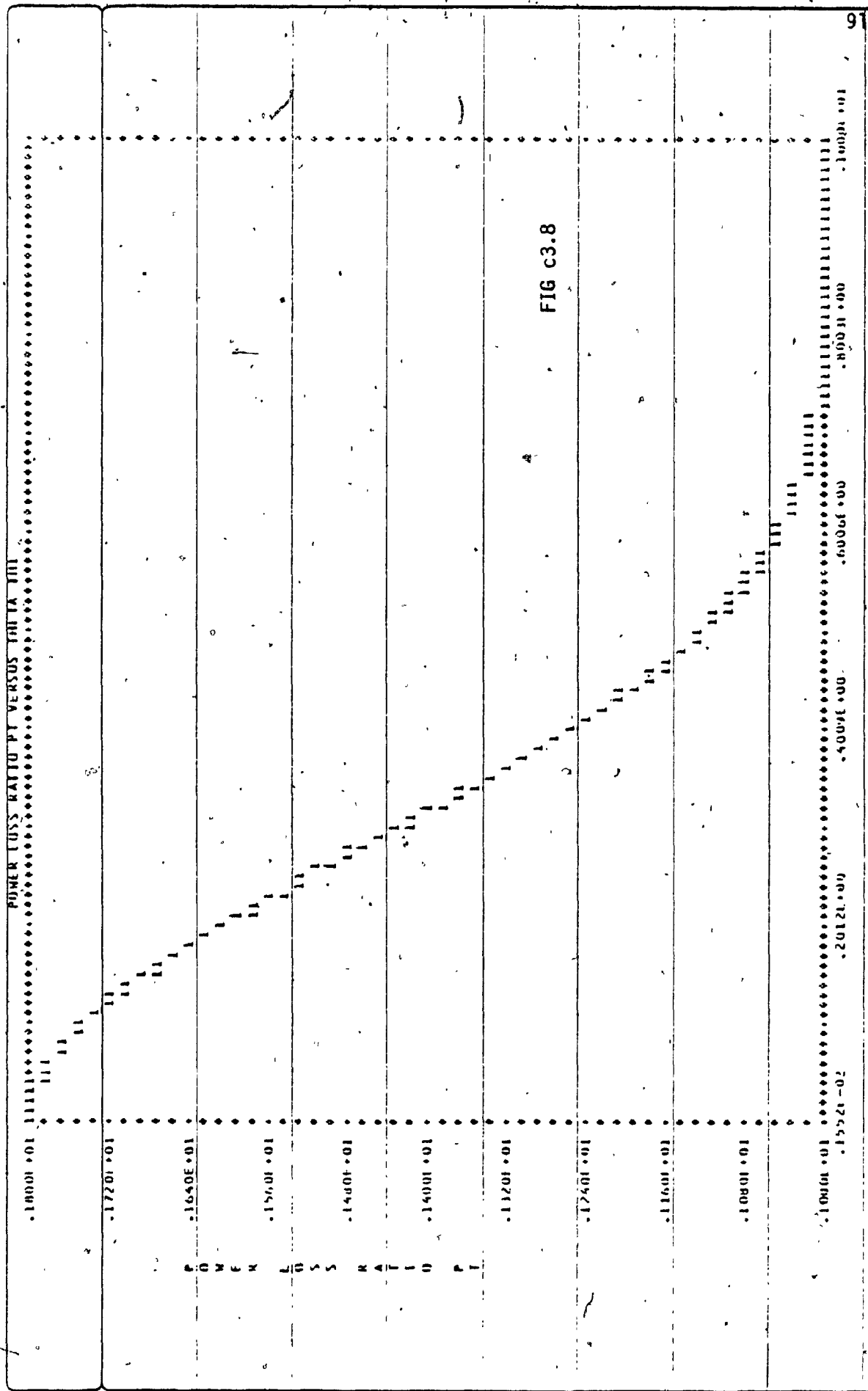
NEW Z1 = .12250913E+01 NEW Z2 = .22360640E+01 NEW Z3 = .40313204E+01

EXTRA PHASE SHIFT PH13 = .21536620E+00 DISTANCE ADDED A3 = .107593+0E+00

TMI

PT

.72153543E-01	.17725467E+01
.55457553E-01	.17773883E+01
.53756567E-01	.17817354E+01
.52055574E-01	.17356327E+01
.45354585E-01	.17390724E+01
.38653594E-01	.17320436E+01
.31752601E-01	.17445546E+01
.25251612E-01	.17955934E+01
.18550621E-01	.17981026E+01
.11349631E-01	.17992433E+01
.51486398E-02	.17998583E+01
.15523513E-02	.17993871E+01
.82533422E-02	.17996350E+01
.14954333E-01	.17998055E+01
.21655324E-01	.17974970E+01
.23356315E-01	.17957126E+01
.35057306E-01	.17934551E+01
.41754297E-01	.17907231E+01
.43459233E-01	.17875357E+01
.55160279E-01	.17834337E+01
.61361270E-01	.17797772E+01
.68562261E-01	.17752230E+01
.75263252E-01	.17702232E+01
.81964242E-01	.17648007E+01
.88665233E-01	.17594471E+01
.95366224E-01	.17526325E+01
.10236722E+00	.17460107E+01
.10376321E+00	.17389440E+01



THETA NORMALIZED

Table 3.3

```

PROGRAM EFFECT (INPUT,OUTPUT)
C THIS PROGRAM OBSERVES THE EFFECT OF SHUNT
C CAPACITANCE ON TRANSMISSION AND REFLECTION
C COEFFICIENTS.
C IT CALCULATES THE NEW VALUES OF TRANSMISSION
C AND REFLECTION COEFFICIENTS.
C IT CALCULATES THE EXTRA PHASE SHIFTS INTRODUCED
C DUE TO CAPACITANCE.
C IT PLOTS THE POWER LOSS RATIO PT FOR THE THIRD
C ORDER BUTTERWORTH AGAINST ELECTRICAL LENGTH
C THETA WHICH IS NORMALIZED TO TH1 WHERE
C TH1=THA/THN, AND THETA IS A FUNCTION OF FREQ.
C THE PASSBAND TOLERANCE IN CASE OF BUTTERWORTH
C IS AK=0.8.
C B IS THE SUSCEPTANCE OF THE CAPACITOR.
C Y4=1/R, WHERE R IS THE TERMINATING RESISTANCE, R=5.
C IT GIVES THE NEW VALUES OF CHARACTERISTIC
C IMPEDANCES EFFECTED BY DISCONTINUITY CAPACITANCE.
C DIMENSION X(161),Y(161,1),A(160),IMAG4(5151)
Y1=0.8161668
Y2=0.4472136
Y3=0.2450479
Y4=0.2
AK=0.8
C=20.0E-12
READ*,W
B=W*C
PRINT 5,W,C,B
5 FORMAT(1H1,/////,10X,*W=*,E15.8,5X,*C=*,E15.8,5X,
  *B=*,E15.8)
T1=SQRT(((2.0*Y1)/Y2)**2/(Y1/Y2+1)**2+(B/Y2)**2)
R1=SQRT(((Y1/Y2-1)**2+(B/Y2)**2)/((Y1/Y2+1)**2
  +(B/Y2)**2))
PRINT 10,T1,R1
10 FORMAT(///,10X,*T1=*,E15.8,5X,*R1=*,E15.8)
YN=((1+R1)*Y2)/(1-R1)
Z1=1/YN
Z2=(5.0)**0.5
Z3=5.0/Z1
PRINT 12,Z1,Z2,Z3
12 FORMAT(///,10X,*NEW Z1=*,E15.8,5X,*NEW Z2=*,
  *E15.8,5X,*NEW Z3=*,E15.8)
AT1=-ATAN((B/Y2)/(Y1/Y2+1))
AT2=-ATAN((B/Y3)/(Y2/Y3+1))
AR3=-ATAN((B/Y4)/(Y3/Y4-1))
$- ATAN((B/Y4)/(Y3/Y4+1))
PHI3=(-AR3 -2*AT1 -2*AT2)
X3=PHI3/2.
PRINT 15,PHI3,X3
15 FORMAT(///,10X,*EXTRA PHASE SHIFT PHI3=*,E15.8,
  *5X,*DISTANCE MOVED X3=*,E15.8,///)

```

```
      THN=(1.6-X3)
      PRINT 20
20     FORMAT(16X,*TH1*,22X,*PT*,//)
      DO 30 I=1,161
      TH=(I-1.0)*0.01
      THA=ABS(TH-X3)
      Q=(COS(THA))*6
      PT=1.0+AK*Q
      TH1=THA/THN
      X(I)=TH1
      Y(I,1)=PT
      PRINT 25,TH1,PT
25     FORMAT(10X,E15.8,10X,E15.8)
30     CONTINUE
      READ 40,(A(I),I=1,160)
40     FORMAT (80A1)
      CALL USPLH (X,Y,161,1,1,161,A,IMAG4,IER)
      STOP
      END
```


W= .19000000E+07 C= .20000000E-10 B= .20000000E-02

T1= .12920442E+01 R1= .29204043E+00

NEW Z1= .12252292E+01 NEW Z2= .22360680E+01 NEW Z3= .40808690E+01

EXTRA PHASE SHIFT PH13= .57906172E-01 DISTANCE MOVED X3= .29903086E-01

TH1

PT

.18396755E-01	.17379973E+01
.12031776E-01	.17991428E+01
.56667950E-02	.17998098E+01
.69818367E-03	.17999971E+01
.70631634E-02	.17997045E+01
.13428143E-01	.17989324E+01
.19793123E-01	.17776821E+01
.26158103E-01	.17959556E+01
.32523082E-01	.17917556E+01
.38883062E-01	.17910857E+01
.45253042E-01	.17879500E+01
.51618021E-01	.17843536E+01
.57983001E-01	.17803022E+01
.64347991E-01	.17758022E+01
.70712961E-01	.17708607E+01
.77077940E-01	.17654856E+01
.83442920E-01	.17596852E+01
.89807900E-01	.17534687E+01
.96172830E-01	.17468458E+01
.10253786E+00	.17398267E+01
.10890284E+00	.17324224E+01
.11526782E+00	.17246443E+01
.12163280E+00	.17165043E+01
.12799778E+00	.17080149E+01
.13436276E+00	.16991391E+01
.14072774E+00	.16900400E+01
.14709272E+00	.16805315E+01
.15345770E+00	.16709279E+01

POWER LOSS RATIO PT VERSUS THETA TH

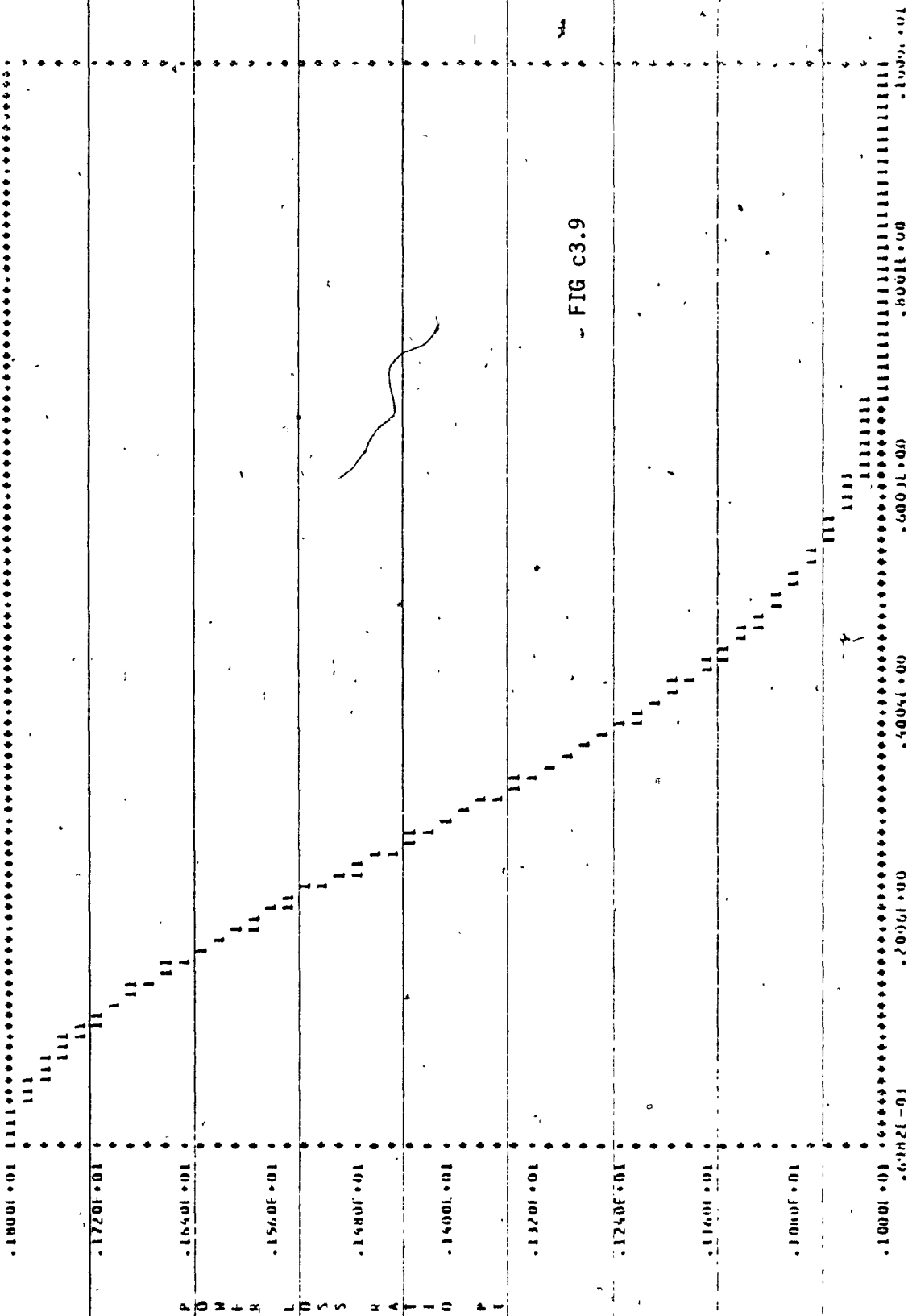


FIG c3.9

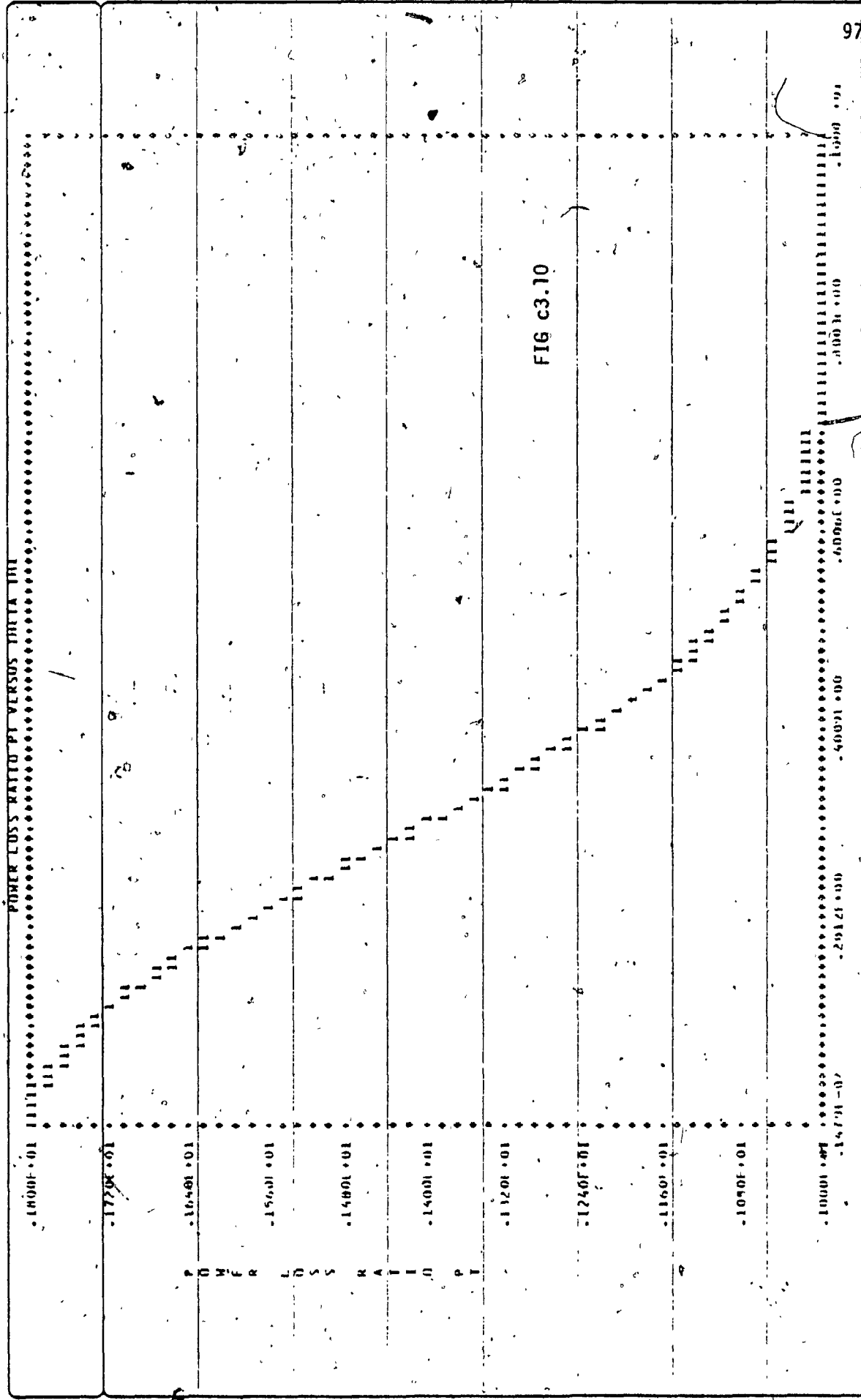
W = .20000000E+01 C = .20000000E-10 E = .40000000E-02

T1 = .12920675E+01 R1 = .29205221E+00

NEW Z1 = .12251977E+01 NEW Z2 = .22346680E+01 NEW Z3 = .40309741E+01

EXTRA PHASE SHIFT PH13 = .11543805E+00 DISTANCE MOVED A3 = .57719027E-01

TH1	PT
.37424457E-01	.17920399E+01
.30940554E-01	.17945315E+01
.24456651E-01	.17965919E+01
.17972748E-01	.17981579E+01
.11488865E-01	.17992468E+01
.50049424E-02	.17998570E+01
.14789605E-02	.17999875E+01
.79625634E-02	.17996341E+01
.14446766E-01	.17988093E+01
.20930609E-01	.17975025E+01
.72741457E-01	.17957193E+01
.33898475E-01	.17934619E+01
.40132373E-01	.17907395E+01
.46866290E-01	.17875481E+01
.53350131E-01	.17848975E+01
.59834086E-01	.17797927E+01
.66317989E-01	.17752400E+01
.72801492E-01	.17702464E+01
.79285793E-01	.17648203E+01
.85759694E-01	.17589707E+01
.92253600E-01	.17527055E+01
.98737503E-01	.17460351E+01
.10522141E+00	.17389699E+01
.11170531E+00	.17315207E+01
.11313921E+00	.17238791E+01
.12467311E+00	.17155171E+01
.13115792E+00	.17069472E+01
.13764092E+00	.16981224E+01



W= .40000000E+09 C= .20000000E-10 B= .80000000E-02

T1= .12921603E+01 R1= .29209929E+00

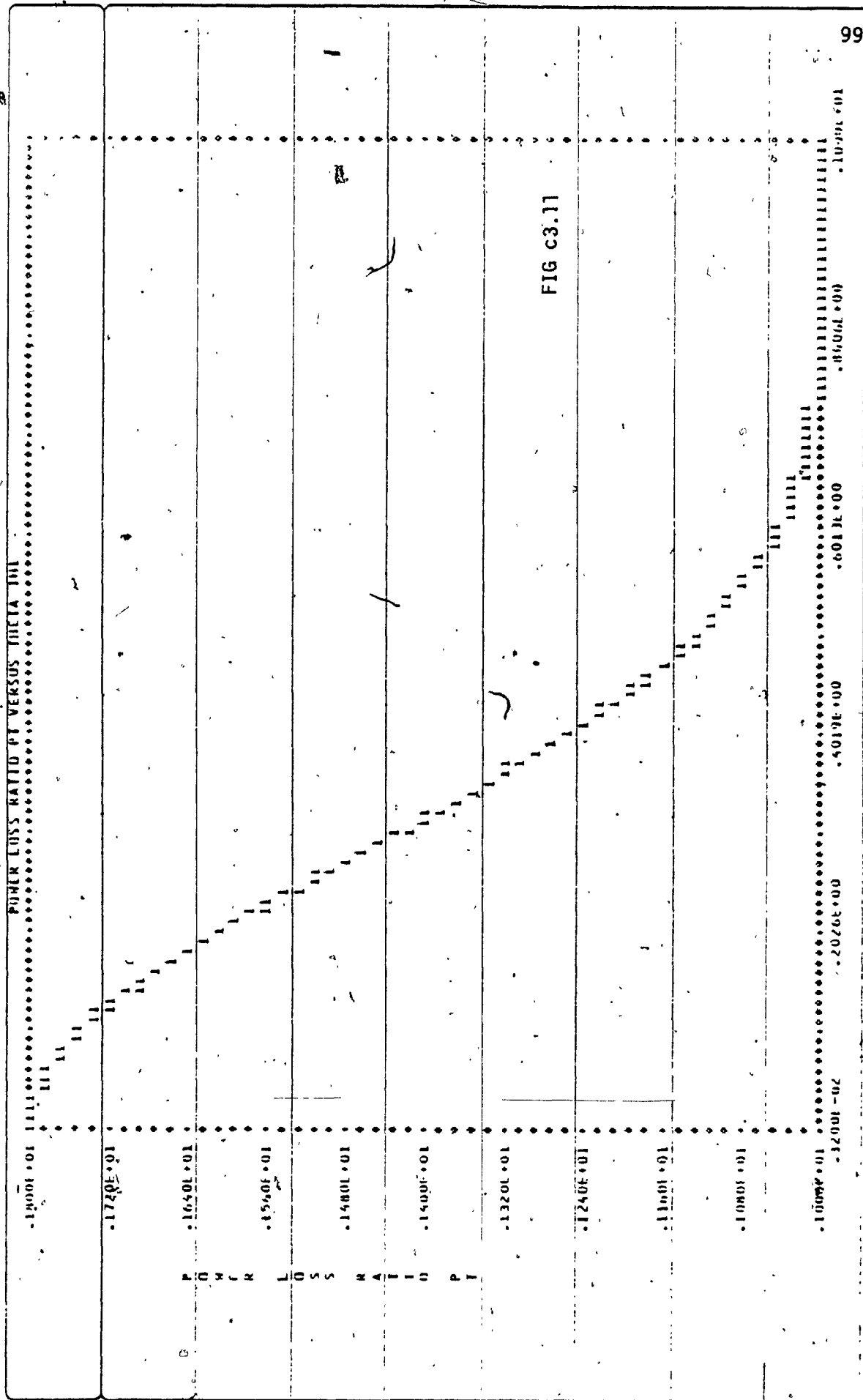
NEW Z1= .12250715E+01 NEW Z2= .22360680E+01 NEW Z3= .40813943E+01

EXTRA PHASE SHIFT PHI3= .22950593E+00 DISTANCE MOVED X3= .11475296E+00

TH1

PT

.77261370E-01	.17589454E+01
.70528983E-01	.17740464E+01
.63796098E-01	.17787086E+01
.57063210E-01	.17929248E+01
.50330323E-01	.17866883E+01
.43597436E-01	.17899930E+01
.36364550E-01	.17928338E+01
.30131663E-01	.17952060E+01
.23398776E-01	.17971060E+01
.16665889E-01	.17985307E+01
.99330028E-02	.17994779E+01
.32001161E-02	.17999458E+01
.35327706E-02	.17999339E+01
.10265657E-01	.17994422E+01
.16998544E-01	.17984715E+01
.23731431E-01	.17970233E+01
.30464317E-01	.17950999E+01
.37197204E-01	.17927044E+01
.43930091E-01	.17938406E+01
.50662977E-01	.17865131E+01
.57395864E-01	.17927271E+01
.64128751E-01	.17734887E+01
.70561637E-01	.17735046E+01
.77594524E-01	.17686322E+01
.84327411E-01	.17631295E+01
.91060297E-01	.17571553E+01
.97793144E-01	.17507637E+01
.10452607E+00	.17439402E+01



THETA NORMAL Z LU - - - - -

W= .50000000E+09 C= .20000000E-19 B= .10000000E-31

T1= .12922300E+01 R1= .23213460E+00

NE+ Z1= .12249770E+01 NE+ Z2= .22360680E+01 NE+ Z3= .40317394E+01

EXTRA PHASE SHIFT PH13= .23362350E+00 DISTANCE MOVED X3= .14231425E+00

TH1

PT

.73006893E-01	.17523599E+01
.91144350E-01	.17586470E+01
.34281305E-01	.17645195E+01
.77419263E-01	.17699633E+01
.70556720E-01	.17749843E+01
.53694177E-01	.17795611E+01
.5331534E-01	.17836900E+01
.49969091E-01	.17873650E+01
.43106548E-01	.17905302E+01
.36244005E-01	.17933304E+01
.29381462E-01	.17956114E+01
.22513919E-01	.17974194E+01
.15655376E-01	.17993751E+01
.87938326E-02	.17996060E+01
.19312895E-02	.17999810E+01
.49312536E-02	.17998761E+01
.11793777E-01	.17992714E+01
.13656340E-01	.17982280E+01
.25513583E-01	.17966874E+01
.32381426E-01	.17946722E+01
.39243969E-01	.17921355E+01
.46106512E-01	.17892316E+01
.52969055E-01	.17858147E+01
.59331598E-01	.17813405E+01
.65894141E-01	.17770152E+01
.73554634E-01	.17728455E+01
.80419227E-01	.17676390E+01
.87281771E-01	.17620039E+01

POWER LOSS RATIO PT VERSUS η_{ETA} 101

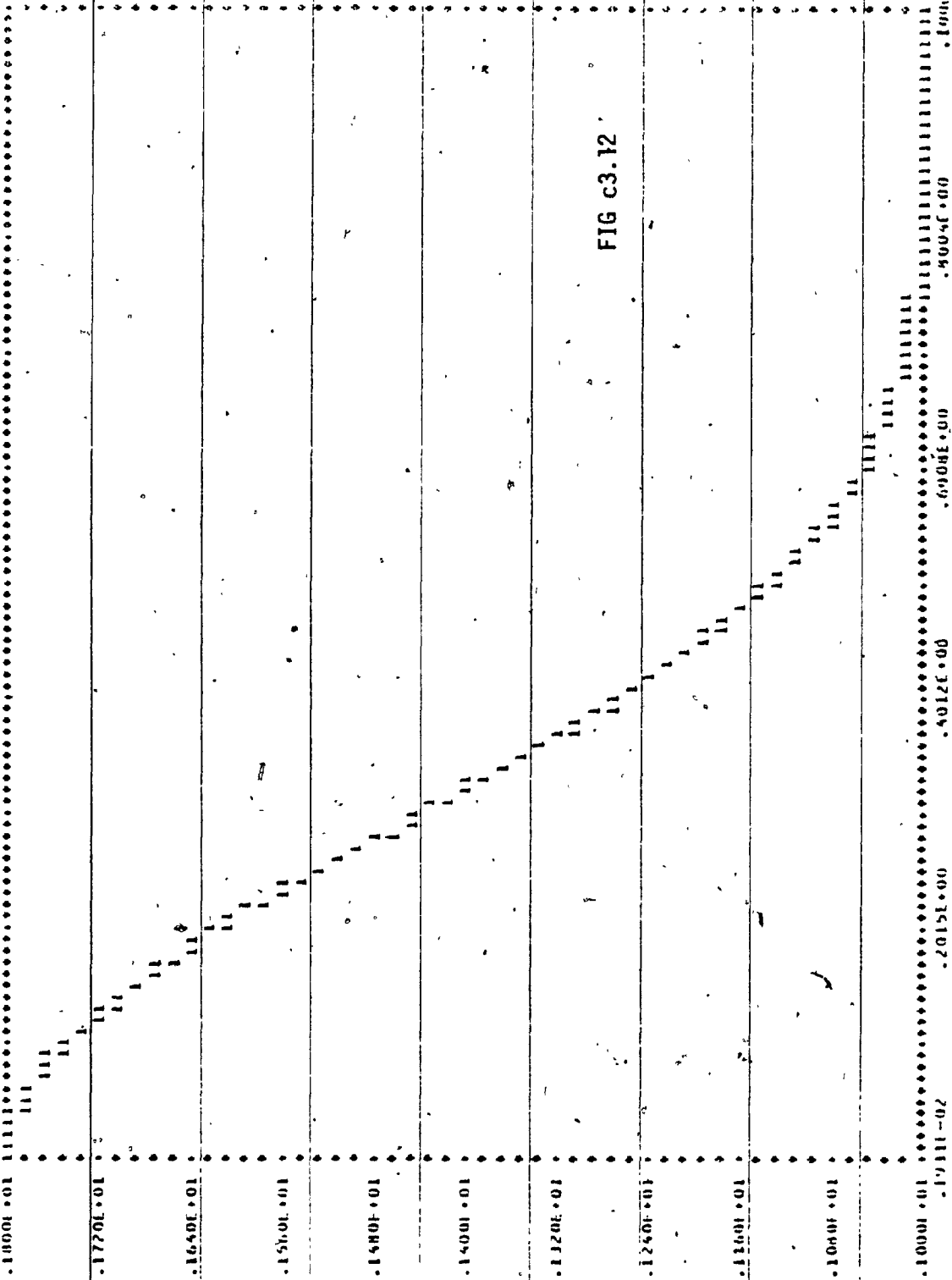


FIG c3.12

DATA NORMALIZED

Table 3.4

```

PROGRAM EFFECT (INPUT,OUTPUT)
C   THIS PROGRAM OBSERVES THE EFFECT OF SHUNT
C   CAPACITANCE ON TRANSMISSION AND REFLECTION
C   COEFFICIENTS.
C   IT CALCULATES THE NEW VALUES OF TRANSMISSION
C   AND REFLECTION COEFFICIENTS.
C   IT CALCULATES THE EXTRA PHASE SHIFTS INTRODUCED
C   DUE TO CAPACITANCE.
C   IT PLOTS THE POWER LOSS RATIO PT FOR THE THIRD
C   ORDER CHEBYSHEV POLYNOMIAL AGAINST ELECTRICAL LENGTH
C   THETA WHICH IS NORMALIZED TO TH1 WHERE
C   TH1=THA/THN, AND THETA IS A FUNCTION OF FREQ.
C   THE POWER LOSS RATIO PT IS PLOTTED FOR
C   ONE FIXED VALUE OF SCALE FACTOR P AND
C   PASSBAND TOLERANCE AK.
C   B IS THE SUSCEPTANCE OF THE CAPACITOR.
C   Y4=1/R, WHERE R IS THE TERMINATING RESISTANCE, R=5.
C   IT GIVES THE NEW VALUES OF CHARACTERISTIC
C   IMPEDANCES EFFECTED BY DISCONTINUITY CAPACITANCE.
C   DIMENSION X(161),Y(161,1),A(160),IMAG4(5151)
Y1=0.5756257
Y2=0.4472136
Y3=0.347448
Y4=0.2
P=0.91
AK=0.19774788157131
C=10.0E-12
READ*,W
B=W*C
PRINT 5,W,C,B
5   FORMAT(1H1,/////,10X,*W=*,E15.8,5X,*C=*,E15.8,5X,
  *B=*,E15.8)
T1=SQRT(((2.0*Y1)/Y2)**2/((Y1/Y2+1)**2+(B/Y2)**2))
R1=SQRT(((Y1/Y2-1)**2+(B/Y2)**2)/((Y1/Y2+1)**2
  +(B/Y2)**2))
PRINT 10,T1,R1
10  FORMAT(///,10X,*T1=*,E15.8,5X,*R1=*,E15.8)
YN=((1+R1)*Y2)/(1-R1)
Z1=1/YN
Z2=(5.0)**0.5
Z3=5.0/Z1
PRINT 12,Z1,Z2,Z3
12  FORMAT(///,10X,*NEW Z1=*,E15.8,5X,*NEW Z2=*,
  *E15.8,5X,*NEW Z3=*,E15.8)
AT1=-ATAN((B/Y2)/(Y1/Y2+1))
AT2=-ATAN((B/Y3)/(Y2/Y3+1))
AR3=-ATAN((B/Y4)/(Y3/Y4-1))
*-ATAN((B/Y4)/(Y3/Y4+1))
PHI3=(-AR3 -2*AT1 -2*AT2)
X3=PHI3/2.
PRINT 15,PHI3,X3
15  FORMAT(///,10X,*EXTRA PHASE SHIFT PHI3=*,E15.8,
  *5X,*DISTANCE MOVED X3=*,E15.8,///)

```

```
      THN=(1.6-X3)
      PRINT 20
20    FORMAT(16X,*TH1*,22X,*PT*,//)
      DO 30 I=1,161
      TH=(I-1.0)*0.01
      THA=ABS(TH-X3)
      Q=COS(THA)/P
      TSCH1=(4.0*Q*Q*Q-3.0*Q)
      TSCH3=TSCH1*TSCH1
      PT=1.0+AK*TSCH3
      TH1=THA/THN
      X(I)=TH1
      Y(I,1)=PT
      PRINT 25,TH1,PT
25    FORMAT(10X,E15.8,10X,E15.8)
30    CONTINUE
      READ 40,(A(I),I=1,160)
40    FORMAT (80A1)
      CALL USPLH (X,Y,161,1,1,161,A,IMAG4,IER)
      STOP
      END
```

W = .10000000E+02 C = .10000000E+10 S = .10000000E+02

T1 = .11255470E+01 R1 = .12554849E+00

NEW Z1 = .17372268E+01 NEW Z2 = .22360680E+01 NEW Z3 = .23781504E+01

EXTRA PHASE SHIFT PH13 = .13080737E+01 DISTANCE MOVED X3 = .55403687E+02

TH1

PT

.41945080E-02	.17997852E+01
.21711445E-02	.17994399E+01
.84467973E-02	.17990405E+01
.14722451E-01	.17972393E+01
.23998104E-01	.17943915E+01
.27273757E-01	.17903547E+01
.33549410E-01	.17857395E+01
.39825064E-01	.17799591E+01
.46100717E-01	.17732292E+01
.52376370E-01	.17655682E+01
.58652023E-01	.17569970E+01
.64927676E-01	.17475389E+01
.71203330E-01	.17372197E+01
.77479983E-01	.17260672E+01
.83754635E-01	.17141113E+01
.90030289E-01	.17013856E+01
.96305942E-01	.16879231E+01
.10258160E+00	.16737603E+01
.10835725E+00	.16589353E+01
.11513290E+00	.16434877E+01
.12140855E+00	.16274585E+01
.12764421E+00	.16108904E+01
.13399986E+00	.15938272E+01
.14023551E+00	.15763137E+01
.14651117E+00	.15583359E+01
.15275682E+00	.15401205E+01
.15906247E+00	.15215350E+01
.16533813E+00	.15025872E+01

PURER LOSS RATIO PT VERSUS THETA THH

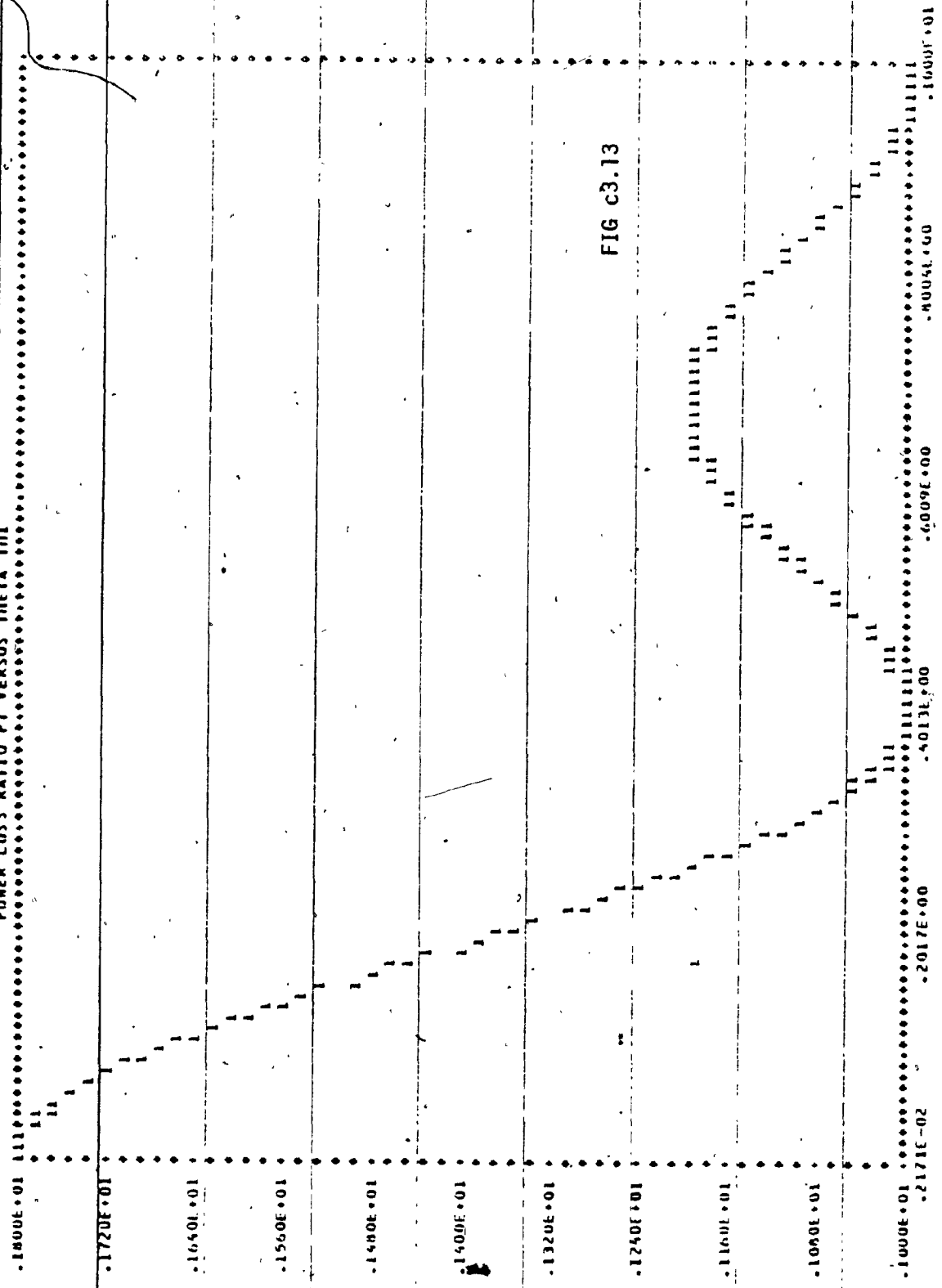


FIG c3.13

THETA NORMALIZED - - - THH

M = .20000000E+09 C = .10000000E-10 B = .20000000E-02

T1 = .11255336E+01 R1 = .12555973E+00

NEW Z1 = .17371371E+01 NEW Z2 = .22360680E+01 NEW Z3 = .23732151E+01

EXTRA PHASE SHIFT PH13 = .26160827E-01 DISTANCE MOVED A3 = .13080414E-01

THL

PT

.82425443E-02	.17991410E+01
.19411277E-02	.17999523E+01
.43603383E-02	.17997595E+01
.10661905E-01	.17985631E+01
.16963422E-01	.17963604E+01
.23264933E-01	.17931754E+01
.29966555E-01	.17889987E+01
.35467971E-01	.17838480E+01
.42169438E-01	.17777371E+01
.48471004E-01	.17706829E+01
.54772521E-01	.17627045E+01
.61074033E-01	.17538236E+01
.67375554E-01	.17440646E+01
.73677071E-01	.17334538E+01
.79978567E-01	.17220200E+01
.86280104E-01	.17097941E+01
.92581520E-01	.16968092E+01
.98883137E-01	.16831002E+01
.10518465E+00	.16687039E+01
.11148617E+00	.16536589E+01
.11773769E+00	.16380053E+01
.12408920E+00	.16217848E+01
.13039072E+00	.16050404E+01
.13669224E+00	.15873152E+01
.14299375E+00	.15701576E+01
.14929527E+00	.15521108E+01
.15559679E+00	.15337227E+01
.16189830E+00	.15150410E+01

POWER LOSS RATIO PT VERSUS THETA TH1

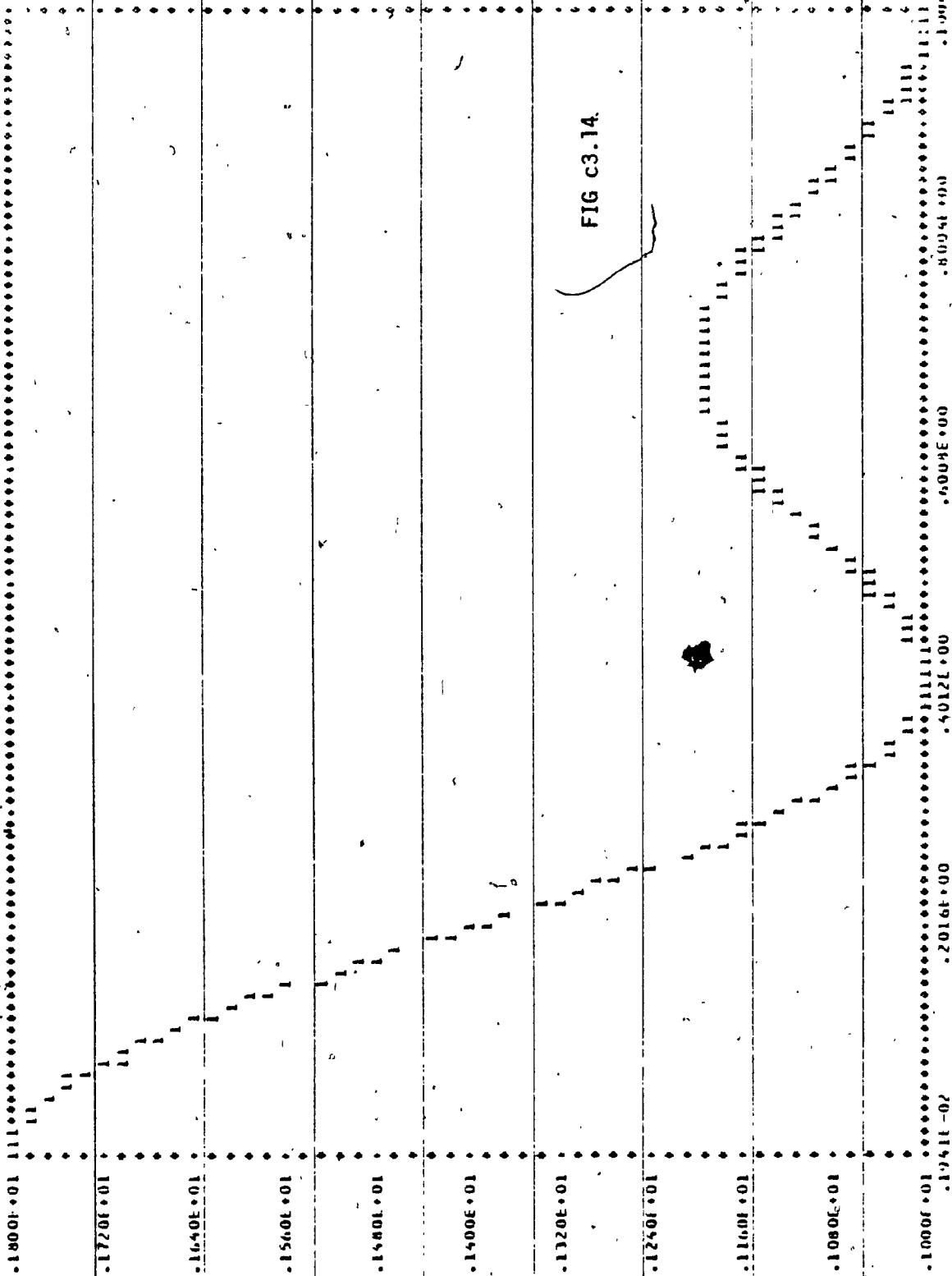


FIG C3.14.

W = .40000000E+07 C = .10000000E-10 G = .40000000E-02

T1 = .11255303E+01 Z1 = .12560463E+00

NEW Z1 = .17370285E+01 NEW Z2 = .22360680E+01 NEW Z3 = .23734770E+01

EXTRA PHASE SHIFT PH13 = .52316475E-01 DISTANCE COVERED X3 = .25153237E-01

TH1	PT
.15620627E-01	.17965687E+01
.10256748E-01	.17986895E+01
.39123693E-02	.17998095E+01
.24410094E-02	.17999259E+01
.87948834E-02	.17990381E+01
.15148757E-01	.17971438E+01
.21502646E-01	.17942630E+01
.27856525E-01	.17903886E+01
.34210404E-01	.17855363E+01
.40564233E-01	.17797192E+01
.46918152E-01	.17729534E+01
.53272041E-01	.17652573E+01
.59625720E-01	.17566517E+01
.65979799E-01	.17471603E+01
.72333679E-01	.17368086E+01
.78687555E-01	.17256249E+01
.85041435E-01	.17136394E+01
.91395314E-01	.17008845E+01
.97749173E-01	.16873945E+01
.10410307E+00	.16732058E+01
.11045695E+00	.16583552E+01
.11641083E+00	.16428856E+01
.12316471E+00	.16268350E+01
.12951359E+00	.16102472E+01
.13587247E+00	.15931659E+01
.14222635E+00	.15756361E+01
.14858022E+00	.15577033E+01
.15493410E+00	.15394157E+01

POWER LOSS RATIO PT VERSUS THETA TH1

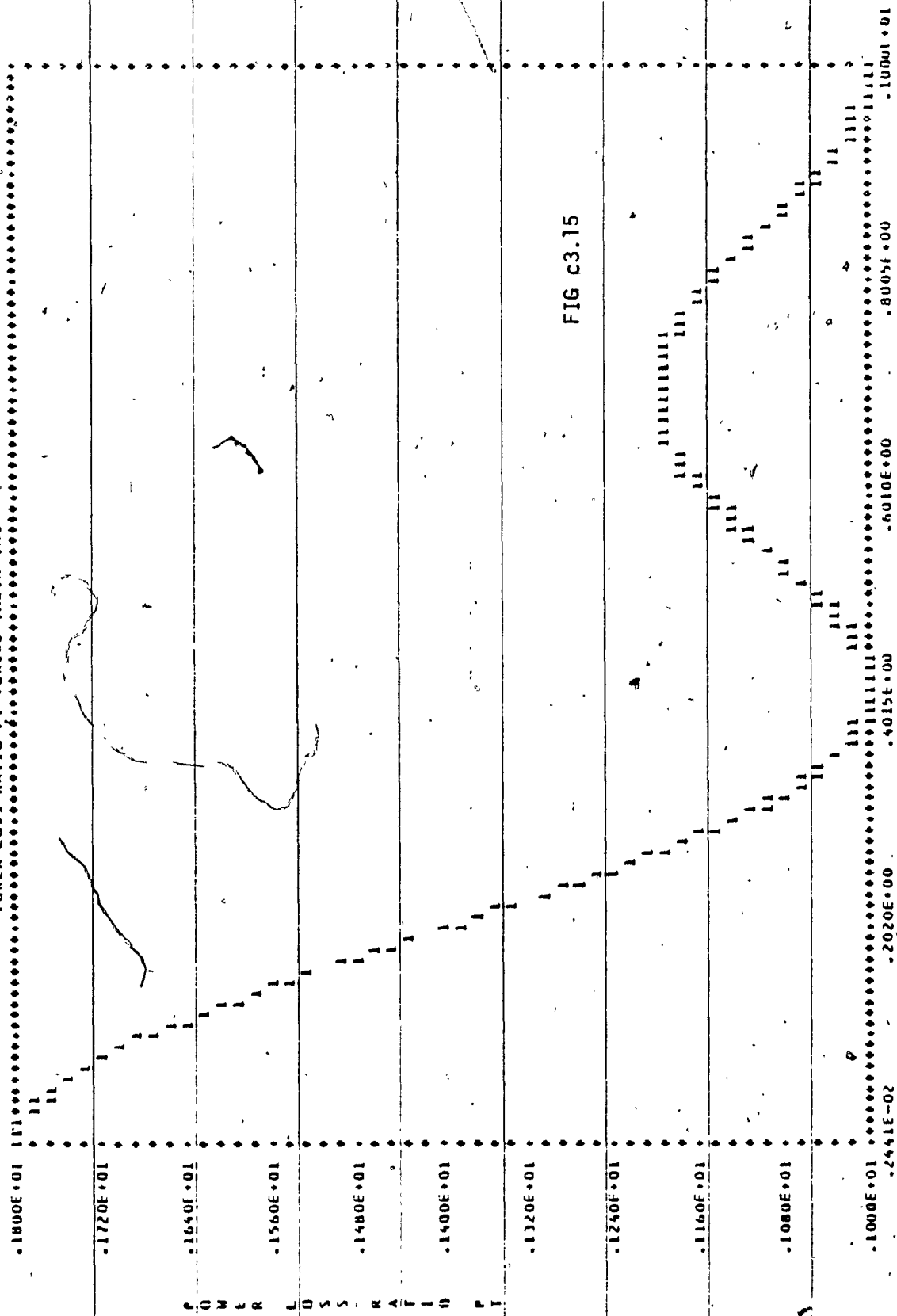


FIG C3.15

W = .50000000E+09 C = .10000000E-10 B = .50000000E-02

T1 = .11256003E+01 R1 = .12583433E+00

NEW Z1 = .17369095E+01 NEW Z2 = .22360680E+01 NEW Z3 = .23746752E+01

EXTRA PHASE SHIFT PH13 = .45390740E-01 DISTANCE MOVED X3 = .32695370E-01

T41

PT

.20860490E-01	.17746441E+01
.14480513E-01	.17974161E+01
.31001293E-02	.17991908E+01
.17197433E-02	.17999615E+01
.46606313E-02	.17997320E+01
.11041012E-01	.17984470E+01
.17421393E-01	.17982619E+01
.23801774E-01	.17930327E+01
.30182154E-01	.17888184E+01
.36562535E-01	.17836303E+01
.42942915E-01	.17774828E+01
.49323296E-01	.17703926E+01
.55703675E-01	.17623791E+01
.62084057E-01	.17534640E+01
.68464417E-01	.17436715E+01
.74844813E-01	.17330286E+01
.81225199E-01	.17215617E+01
.87605573E-01	.17093030E+01
.93985960E-01	.16962945E+01
.10036614E+00	.16825583E+01
.10674672E+00	.16681364E+01
.11312710E+00	.16530672E+01
.11950748E+00	.16373910E+01
.12588736E+00	.16211495E+01
.13226324E+00	.16043353E+01
.13864862E+00	.15871440E+01
.14502900E+00	.15694596E+01
.15140933E+00	.15514084E+01

POWER LOSS RATIO PT VERSUS THETA TH1

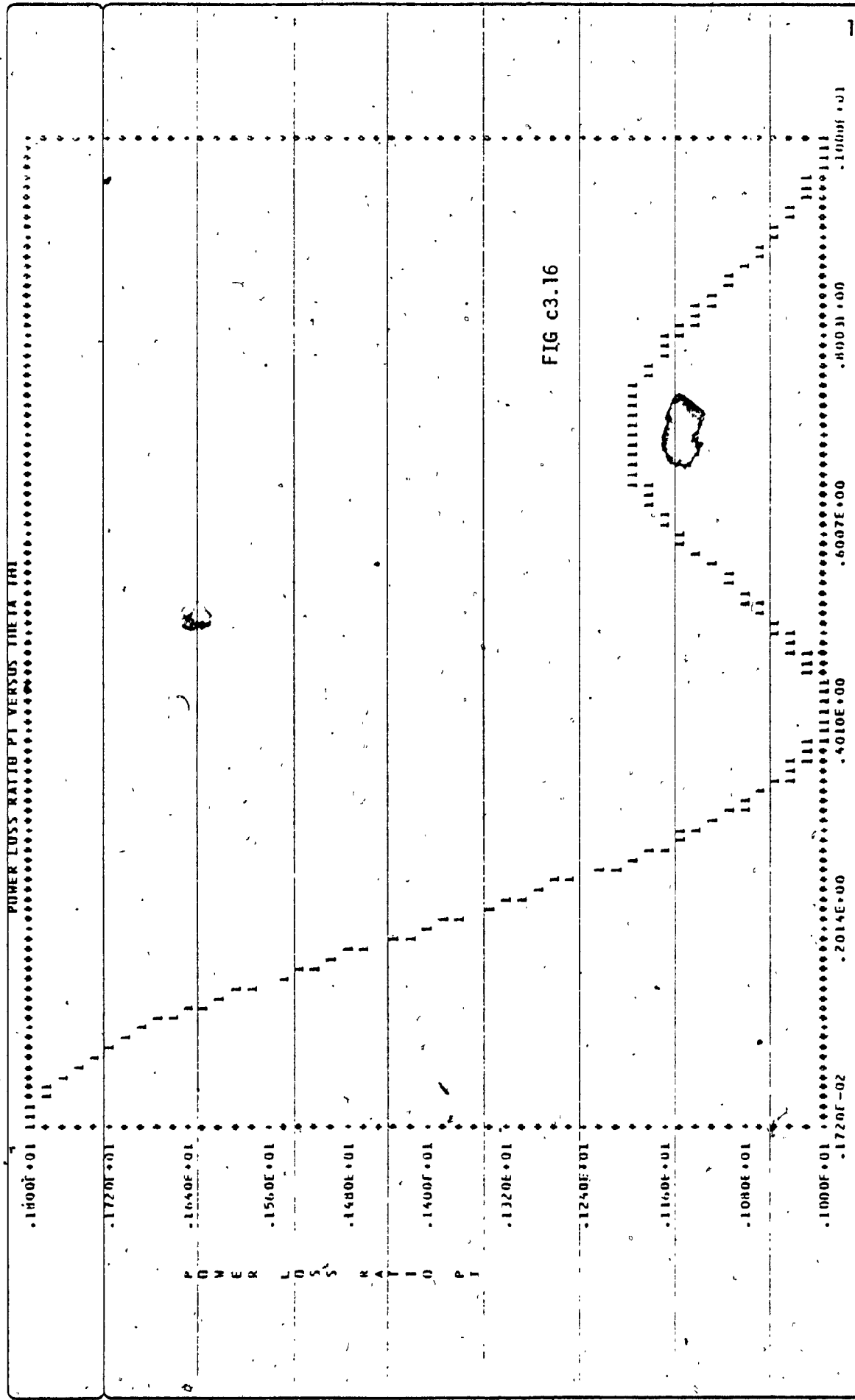


FIG c3.16

THETA NORMALIZED

Table 3.5

```

PROGRAM EFFECT (INPUT,OUTPUT)
C THIS PROGRAM OBSERVES THE EFFECT OF SHUNT
C CAPACITANCE ON TRANSMISSION AND REFLECTION
C COEFFICIENTS.
C IT CALCULATES THE NEW VALUES OF TRANSMISSION
C AND REFLECTION COEFFICIENTS.
C IT CALCULATES THE EXTRA PHASE SHIFTS INTRODUCED
C DUE TO CAPACITANCE.
C IT PLOTS THE POWER LOSS RATIO PT FOR THE THIRD
C ORDER CHEBYSHEV POLYNOMIAL AGAINST ELECTRICAL LENGTH
C THETA WHICH IS NORMALIZED TO TH1 WHERE
C TH1=THA/THN, AND THETA IS A FUNCTION OF FREQ.
C THE POWER LOSS RATIO PT IS PLOTTED FOR
C ONE FIXED VALUE OF SCALE FACTOR P AND
C PASSBAND TOLERANCE AK.
C B IS THE SUSCEPTANCE OF THE CAPACITOR.
C Y4=1/R, WHERE R IS THE TERMINATING RESISTANCE, R=5.
C IT GIVES THE NEW VALUES OF CHARACTERISTIC
C IMPEDANCES EFFECTED BY DISCONTINUITY CAPACITANCE.
C DIMENSION X(161),Y(161,1),A(160),IMAG4(5151)
Y1=0.5756257
Y2=0.4472136
Y3=0.347448
Y4=0.2
P=0.91
AK=0.19774788157131
C=15.0E-12
READ*,W
B=W*C
PRINT 5,W,C,B
5 FORMAT(1H1,/////,10X,*W=*,E15.8,5X,*C=*,E15.8,5X,
*$B=*,E15.8)
T1=SQRT(((2.0*Y1)/Y2)**2/((Y1/Y2+1)**2+(B/Y2)**2)
R1=SQRT(((Y1/Y2-1)**2+(B/Y2)**2)/((Y1/Y2+1)**2
$+(B/Y2)**2))
PRINT 10,T1,R1
10 FORMAT(///,10X,*T1=*,E15.8,5X,*R1=*,E15.8)
YN=((1+R1)*Y2)/(1-R1)
Z1=1/YN
Z2=(5.0)**0.5
Z3=5.0/Z1
PRINT 12,Z1,Z2,Z3
12 FORMAT(///,10X,*NEW Z1=*,E15.8,5X,*NEW Z2=*,
*$E15.8,5X,*NEW Z3=*,E15.8)
AT1=-ATAN((B/Y2)/(Y1/Y2+1))
AT2=-ATAN((B/Y3)/(Y2/Y3+1))
AR3=-ATAN((B/Y4)/(Y3/Y4-1))
$- ATAN((B/Y4)/(Y3/Y4+1))
PHI3=(-AR3 -2*AT1 -2*AT2)
X3=PHI3/2.
PRINT 15,PHI3,X3
15 FORMAT(///,10X,*EXTRA PHASE SHIFT PHI3=*,E15.8,
*$5X,*DISTANCE MOVED X3=*,E15.8,///)

```

```
THN=(1.6-X3)
PRINT 20
20  FORMAT(16X,*TH1*,22X,*PT*,//)
   DO 30 I=1,161
   TH=(I-1.0)*0.01
   THA=ABS(TH-X3)
   Q=COS(THA)/P
   TSCH1=(4.0*Q*Q*Q-3.0*Q)
   TSCH3=TSCH1*TSCH1
   PT=1.0+AK*TSCH3
   TH1=THA/THN
   X(I)=TH1
   Y(I,1)=PT
25  PRINT 25,TH1,PT
30  FORMAT(10X,E15.8,10X,E15.8)
   CONTINUE
40  READ 40,(A(I),I=1,160)
   FORMAT (80A1)
   CALL USPLH (X,Y,161,1,1,161,A,IMAG4,IER)
   STOP
   END
```

A = .10000000E+00 C = .15000000E-10 B = .15000000E-02

T1 = .11255497E+01 R1 = .12555313E+00

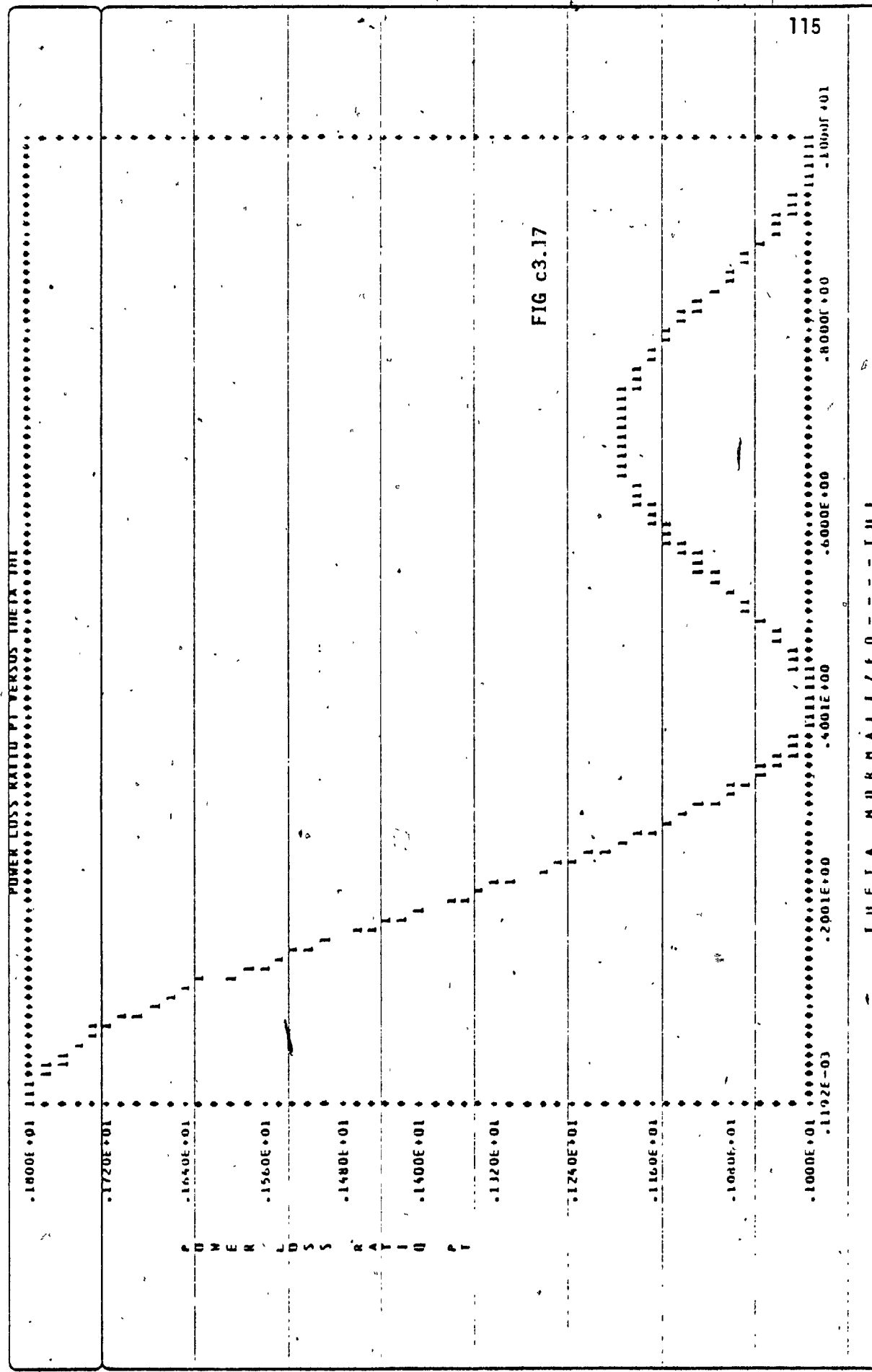
NEW Z1 = .17372103E+01 NEW Z2 = .22300880E+01 NEW Z3 = .23731773E+01

EXTRA PHASE SHIFT PH13 = .19620904E-01 DISTANCE MOVED X3 = .93104513E-02

T1L

PT

.61693601E-02	.17395167E+01
.11919844E-03	.17999994E+01
.64077567E-02	.17994787E+01
.12696315E-01	.17979547E+01
.16984874E-01	.17954320E+01
.25273432E-01	.17919176E+01
.31561991E-01	.17874210E+01
.37850549E-01	.17819546E+01
.44139108E-01	.17753333E+01
.50427665E-01	.17681746E+01
.56716225E-01	.17598986E+01
.63004793E-01	.17507279E+01
.69293342E-01	.17406873E+01
.75581900E-01	.17298040E+01
.81870459E-01	.17181077E+01
.88159017E-01	.17056299E+01
.94447576E-01	.16924043E+01
.10073613E+00	.16784664E+01
.10702469E+00	.16638537E+01
.11331329E+00	.16486052E+01
.11960131E+00	.16327615E+01
.12589037E+00	.16163650E+01
.13217393E+00	.15994533E+01
.13846749E+00	.15820875E+01
.14475604E+00	.15642968E+01
.15104460E+00	.15461332E+01
.15733316E+00	.15276434E+01
.16362172E+00	.15088765E+01



W = .20000000E+09 C = .15000000E-10 B = .10000000E-02

T1 = .11255647E+01 R1 = .12557340E+00

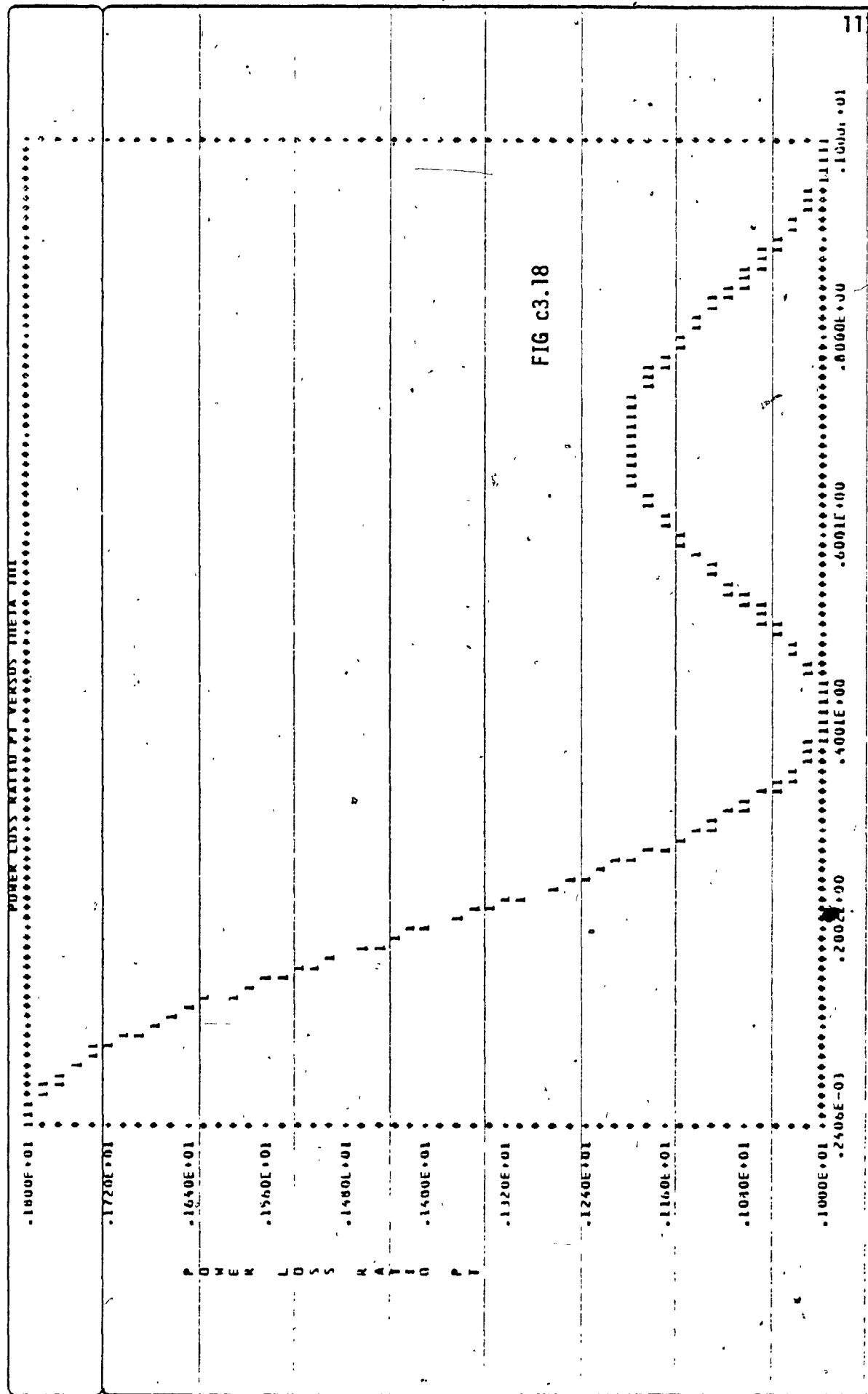
NEW Z1 = .17371210E+01 NEW Z2 = .22360680E+01 NEW Z3 = .29783257E+01

EXTRA PHASE SHIFT PH13 = .09239622E-01 DISTANCE MOVED A3 = .19619311E-01

TH1

PT

.12414615E-01	.17980634E+01
.60870232E-02	.17995353E+01
.24056809E-03	.17999993E+01
.65681594E-02	.17994590E+01
.12895751E-01	.17979159E+01
.19223342E-01	.17953742E+01
.25550933E-01	.17918410E+01
.31173525E-01	.17873253E+01
.38206115E-01	.17813411E+01
.44533707E-01	.17754017E+01
.50861299E-01	.17680254E+01
.57189390E-01	.17597321E+01
.63516482E-01	.17505445E+01
.69844073E-01	.17404876E+01
.76171664E-01	.17295886E+01
.82499256E-01	.17178771E+01
.88826847E-01	.17053847E+01
.95154438E-01	.16921451E+01
.10148203E+00	.16781940E+01
.10730962E+00	.16635685E+01
.11413721E+00	.16483086E+01
.12046490E+00	.16324540E+01
.12579239E+00	.16160473E+01
.13111999E+00	.15991313E+01
.13744753E+00	.15817521E+01
.14377517E+00	.15639539E+01
.15010276E+00	.15457336E+01
.15643035E+00	.15272885E+01
.16275794E+00	.15085153E+01
.16908553E+00	.14895154E+01



M = .40000000E+09 C = .15000000E-10 S = .50000000E+12

T1 = .11256247E+01 R1 = .12567955E+00

NEW Z1 = .17367542E+01 NEW Z2 = .22350880E+01 NEW Z3 = .23799170E+01

EXTRA PHASE SHIFT PH13 = .78461774E-01 DISTANCE MOVED X1 = .39230337E-01

TH1

PT

.25135612E-01	.17922972E+01
.13723515E-01	.17957169E+01
.12321417E-01	.17981441E+01
.59143194E-02	.17995721E+01
.49277913E-03	.17999970E+01
.68998757E-02	.17994177E+01
.13306973E-01	.17978357E+01
.19714071E-01	.17952553E+01
.26121168E-01	.17916837E+01
.32528266E-01	.17871305E+01
.38435364E-01	.17816084E+01
.45342461E-01	.17751322E+01
.51749559E-01	.17677198E+01
.58156656E-01	.17593913E+01
.64563754E-01	.17501694E+01
.70970851E-01	.17400792E+01
.77377949E-01	.17291481E+01
.83785047E-01	.17174056E+01
.90192144E-01	.17048835E+01
.96599242E-01	.16916156E+01
.10300634E+00	.16776376E+01
.10941344E+00	.16629869E+01
.11582053E+00	.16477023E+01
.12222763E+00	.16318260E+01
.12863473E+00	.16153935E+01
.13504133E+00	.15984642E+01
.14144892E+00	.15810674E+01
.14785602E+00	.15632538E+01

POWER LOSS RATIO PL VERSUS THEJA THI

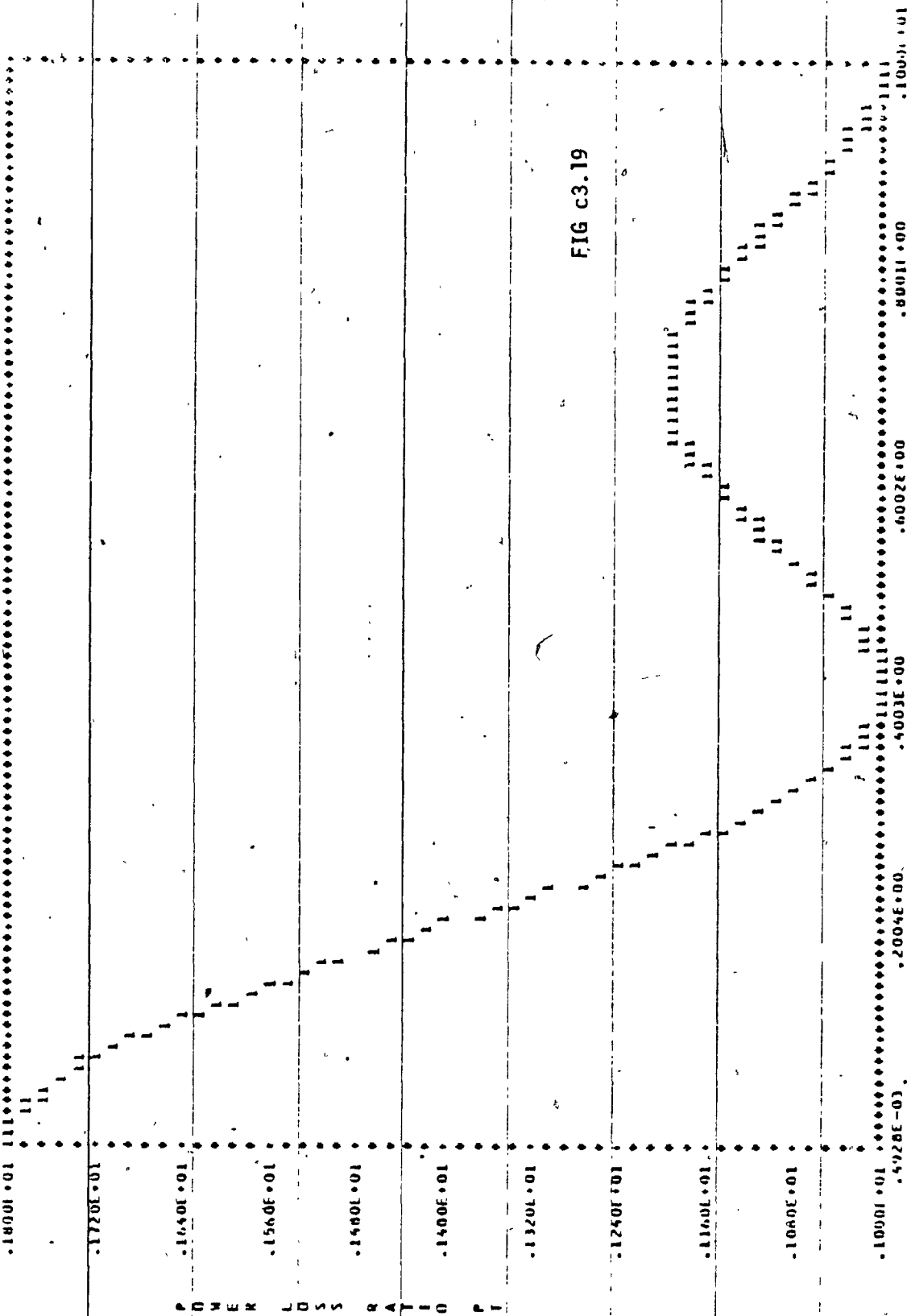


FIG c3.19

THEJA NORMALIZED

A = .50000000E+09 C = .15000000E-10 B = .75000000E-02

TL = .11256697E+01 R1 = .12575531E+00

NEW Z1 = .17364968E+01 NEW Z2 = .22360630E+01 NEW Z3 = .23733603E+01

EXTRA PHASE SHIFT PH13 = .98060861E-01 DISTANCE MOVED X1 = .49910410E-01

TH1	PT
.31612761E-01	.17879922E+01
.25165191E-01	.17923755E+01
.13717601E-01	.17957754E+01
.12270022E-01	.17981826E+01
.53224420E-02	.17995905E+01
.62513779E-03	.17999953E+01
.70727175E-02	.17993953E+01
.13520297E-01	.17977937E+01
.19967877E-01	.17951934E+01
.25415457E-01	.17916020E+01
.32863037E-01	.17870293E+01
.39310615E-01	.17814879E+01
.45758195E-01	.17749923E+01
.52205776E-01	.17675618E+01
.58653356E-01	.17592152E+01
.65100935E-01	.17499756E+01
.71548515E-01	.17399683E+01
.77996095E-01	.17299206E+01
.84443675E-01	.17171621E+01
.90891254E-01	.17046248E+01
.97338834E-01	.16913423E+01
.10378641E+00	.16773904E+01
.11023399E+00	.16625666E+01
.11668157E+00	.16473902E+01
.12312915E+00	.16315020E+01
.12957673E+00	.16150640E+01
.13602431E+00	.15981199E+01
.14247139E+00	.15807143E+01

POWER LOSS RATIO PT VERSUS THETA TH1

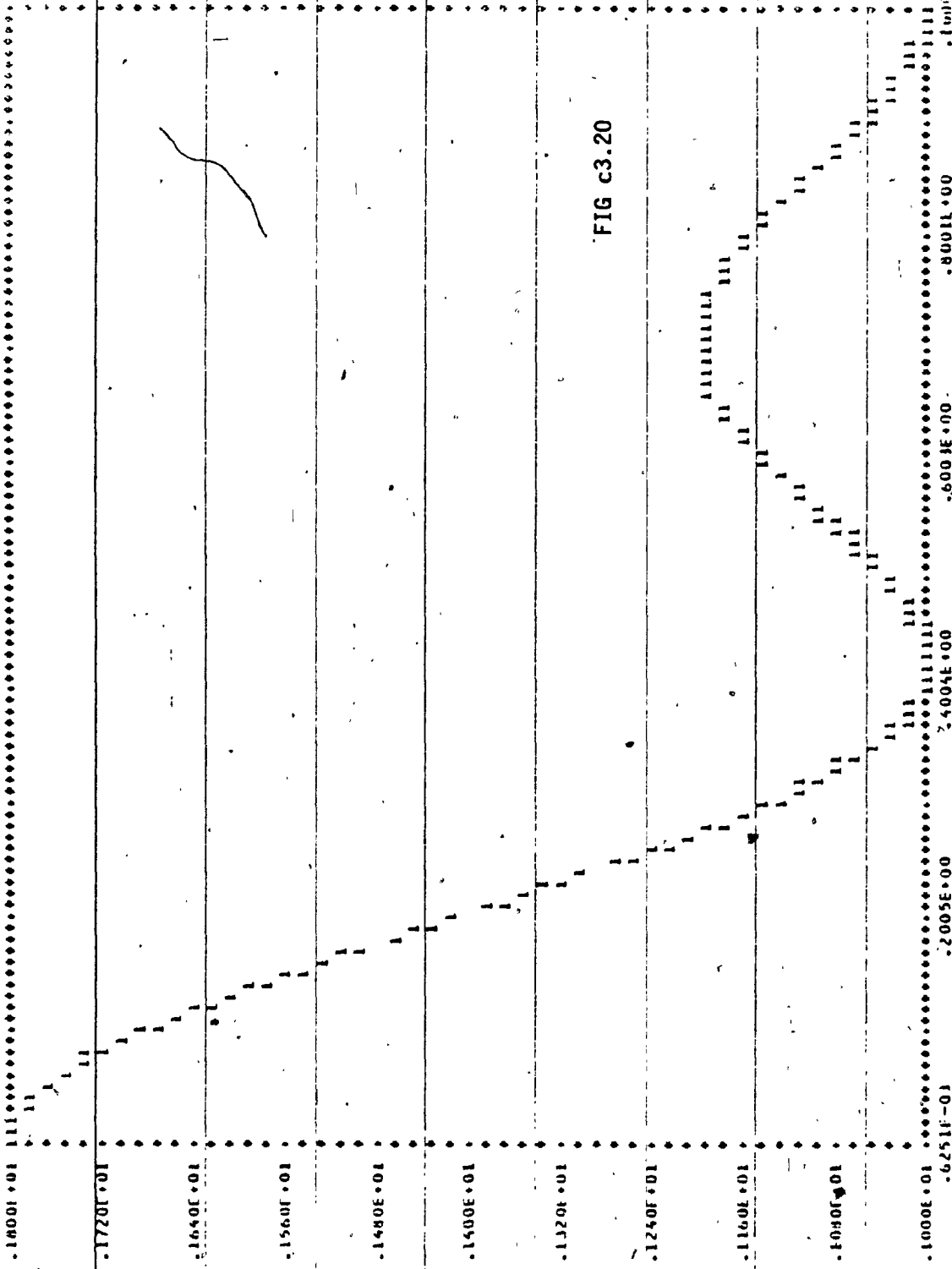


FIG c3.20

Table 3.6

```

PROGRAM EFFECT (INPUT,OUTPUT)
C   THIS PROGRAM OBSERVES THE EFFECT OF SHUNT
C   CAPACITANCE ON TRANSMISSION AND REFLECTION
C   COEFFICIENTS.
C   IT CALCULATES THE NEW VALUES OF TRANSMISSION
C   AND REFLECTION COEFFICIENTS.
C   IT CALCULATES THE EXTRA PHASE SHIFTS INTRODUCED
C   DUE TO CAPACITANCE.
C   IT PLOTS THE POWER LOSS RATIO PT FOR THE THIRD
C   ORDER CHEBYSHEV POLYNOMIAL AGAINST ELECTRICAL LENGTH.
C   THETA WHICH IS NORMALIZED TO TH1 WHERE
C   TH1=THA/THN, AND THETA IS A FUNCTION OF FREQ.
C   THE POWER LOSS RATIO PT IS PLOTTED FOR
C   ONE FIXED VALUE OF SCALE FACTOR P AND
C   PASSBAND TOLERANCE AK.
C   B IS THE SUSCEPTANCE OF THE CAPACITOR.
C   Y4=1/R, WHERE R IS THE TERMINATING RESISTANCE, R=5.
C   IT GIVES THE NEW VALUES OF CHARACTERISTIC
C   IMPEDANCES EFFECTED BY DISCONTINUITY CAPACITANCE.
DIMENSION X(161),Y(161,1),A(160),IMAG4(5151)
Y1=0.5756257
Y2=0.4472136
Y3=0.347448
Y4=0.2
P=0.91
AK=0.19774788157131
C=20.0E-12
READ*,W
B=W*C
PRINT 5,W,C,B
5  FORMAT(1H1,/////,10X,*W=*,E15.8,5X,*C=*,E15.8,5X,
$*B=*,E15.8)
T1=SQRT(((2.0*Y1)/Y2)**2/(Y1/Y2+1)**2+(B/Y2)**2)
R1=SQRT(((Y1/Y2-1)**2+(B/Y2)**2)/((Y1/Y2+1)**2
$+(B/Y2)**2))
PRINT 10,T1,R1
10  FORMAT(///,10X,*T1=*,E15.8,5X,*R1=*,E15.8)
YN=((1+R1)*Y2)/(1-R1)
Z1=1/YN
Z2=(5.0)**0.5
Z3=5.0/Z1
PRINT 12,Z1,Z2,Z3
12  FORMAT(///,10X,*NEW Z1=*,E15.8,5X,*NEW Z2=*,
$E15.8,5X,*NEW Z3=*,E15.8)
AT1=-ATAN((B/Y2)/(Y1/Y2+1))
AT2=-ATAN((B/Y3)/(Y2/Y3+1))
AR3=-ATAN((B/Y4)/(Y3/Y4-1))
$- ATAN((B/Y4)/(Y3/Y4+1))
PHI3=(-AR3 -2*AT1 -2*AT2)
X3=PHI3/2.
PRINT 15,PHI3,X3
15  FORMAT(///,10X,*EXTRA PHASE SHIFT PHI3=*,E15.8,
$5X,*DISTANCE MOVED X3=*,E15.8,///)

```

```
      THN=(1.6-X3)
      PRINT 20
20    FORMAT(16X,*TH1*,22X,*PT*,//)
      DO 30 I=1,161
      TH=(I-1.0)*0.01
      THA=ABS(TH-X3)
      Q=COS(THA)/P
      TSCH1=(4.0*Q*Q*Q-3.0*Q)-
      TSCH3=TSCH1*TSCH1
      PT=1.0+AK*TSCH3
      TH1=THA/THN
      X(I)=TH1
      Y(I,1)=PT
      PRINT 25/TH1,PT \
25    FORMAT(10X,E15.8,10X,E15.8)
30    CONTINUE
      READ 40,(A(I),I=1,160)
40    FORMAT (80A1)
      CALL USPLH (X,Y,161,1,1,161,A,IMAG4,IER)
      STOP
      END
```

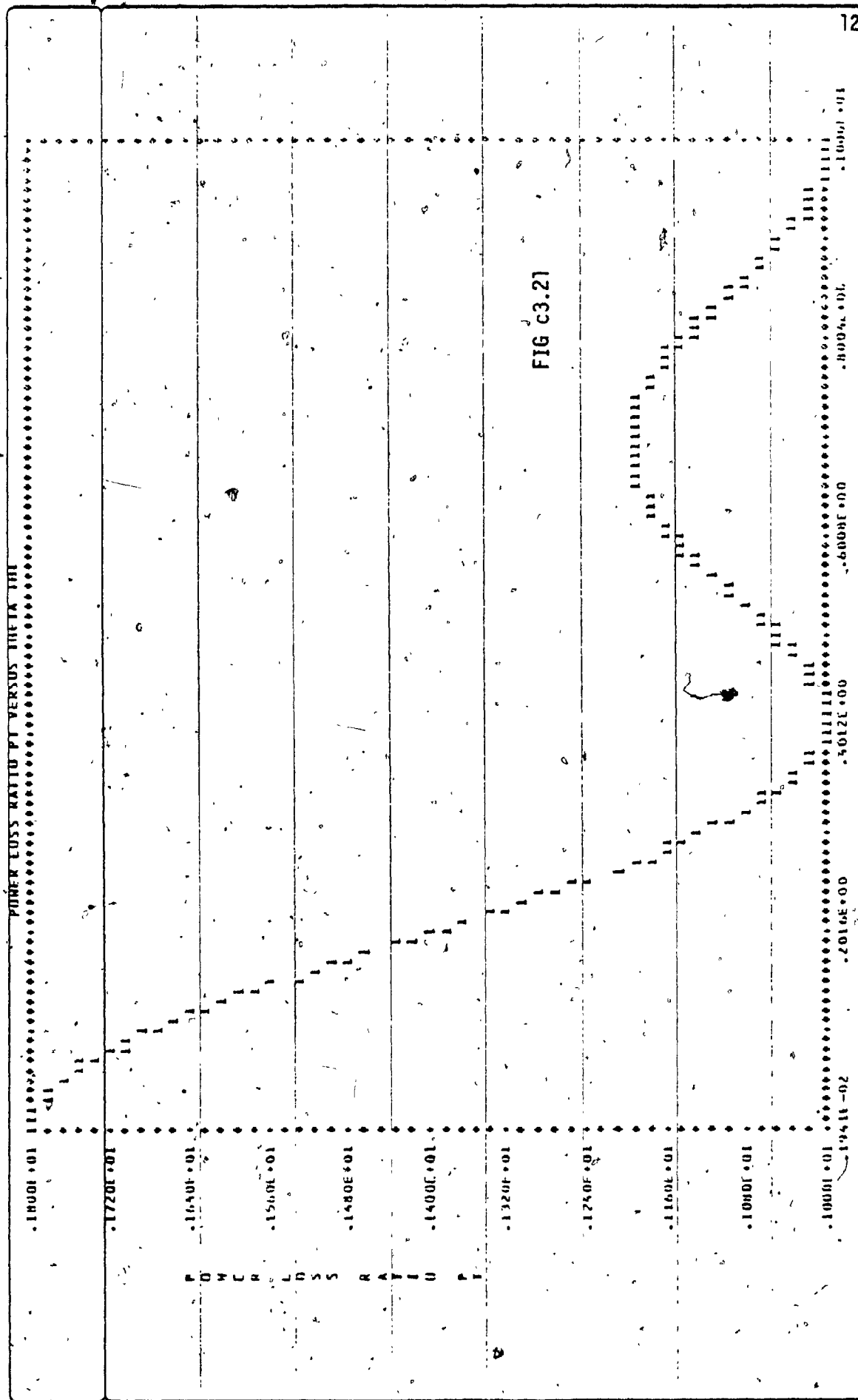
W = .10000000E+00 C = .20000000E-10 S = .20000000E-02

T1 = .11255536E+01 R1 = .12555973E+00

NEW Z1 = .17371871E+01 NEW Z2 = .22360640E+01 NEW Z3 = .23782151E+01

EXTRA PHASE SHIFT PH13 = .26160827E-01 DISTANCE MOVED X3 = .13080414E-01

TH1	PT
.82426443E-02	.17491410E+01
.19411277E-02	.17999523E+01
.43603388E-02	.17997595E+01
.10661905E-01	.17985631E+01
.16963422E-01	.17963664E+01
.23264938E-01	.17931754E+01
.29586455E-01	.17889987E+01
.35867971E-01	.17838480E+01
.42169488E-01	.17777371E+01
.48471004E-01	.17706829E+01
.54772521E-01	.17627045E+01
.61074038E-01	.17538236E+01
.67375554E-01	.17440646E+01
.73677071E-01	.17334538E+01
.79978587E-01	.17220200E+01
.86280104E-01	.17097941E+01
.92581620E-01	.16968092E+01
.98883137E-01	.16831002E+01
.10518455E+00	.16687037E+01
.11148617E+00	.16536539E+01
.11778769E+00	.16380053E+01
.12408920E+00	.16217848E+01
.13039072E+00	.16050404E+01
.13669224E+00	.15878162E+01
.14299375E+00	.15701576E+01
.14929527E+00	.15521108E+01
.15559679E+00	.15337227E+01
.16189830E+00	.15150410E+01



A = .20000000E+09 C = .20000000E+10 d = .40000000E-02

T1 = .11255803E+01 R1 = .12560463E+00

NEW Z1 = .17370285E+01 NEW Z2 = .22160680E+01 NEW Z3 = .29794790E+01

EXTRA PHASE SHIFT PH13 = .52316475E-01 DISTANCE MOVED X3 = .26153237E-01

TH1

PT

.16620627E-01	.17965687E+01
.10266744E-01	.17966895E+01
.39123695E-02	.17998095E+01
.24410094E-02	.17999259E+01
.37948884E-02	.17990381E+01
.15148767E-01	.17971488E+01
.21302646E-01	.17942630E+01
.27356525E-01	.17903886E+01
.34210404E-01	.17855363E+01
.40564283E-01	.17797192E+01
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.53272041E-01	.17692573E+01
.59525420E-01	.17566517E+01
.65979799E-01	.17471603E+01
.72333678E-01	.17366036E+01
.78687556E-01	.17256249E+01
.85041435E-01	.17136394E+01
.91395314E-01	.17008845E+01
.97749173E-01	.16873945E+01
.10410307E+00	.16732058E+01
.11045695E+00	.16583562E+01
.11581083E+00	.16429856E+01
.12316471E+00	.16268350E+01
.12951959E+00	.16102472E+01
.13587247E+00	.15931659E+01
.14222635E+00	.15756361E+01
.14858022E+00	.15577038E+01
.15493410E+00	.15394157E+01

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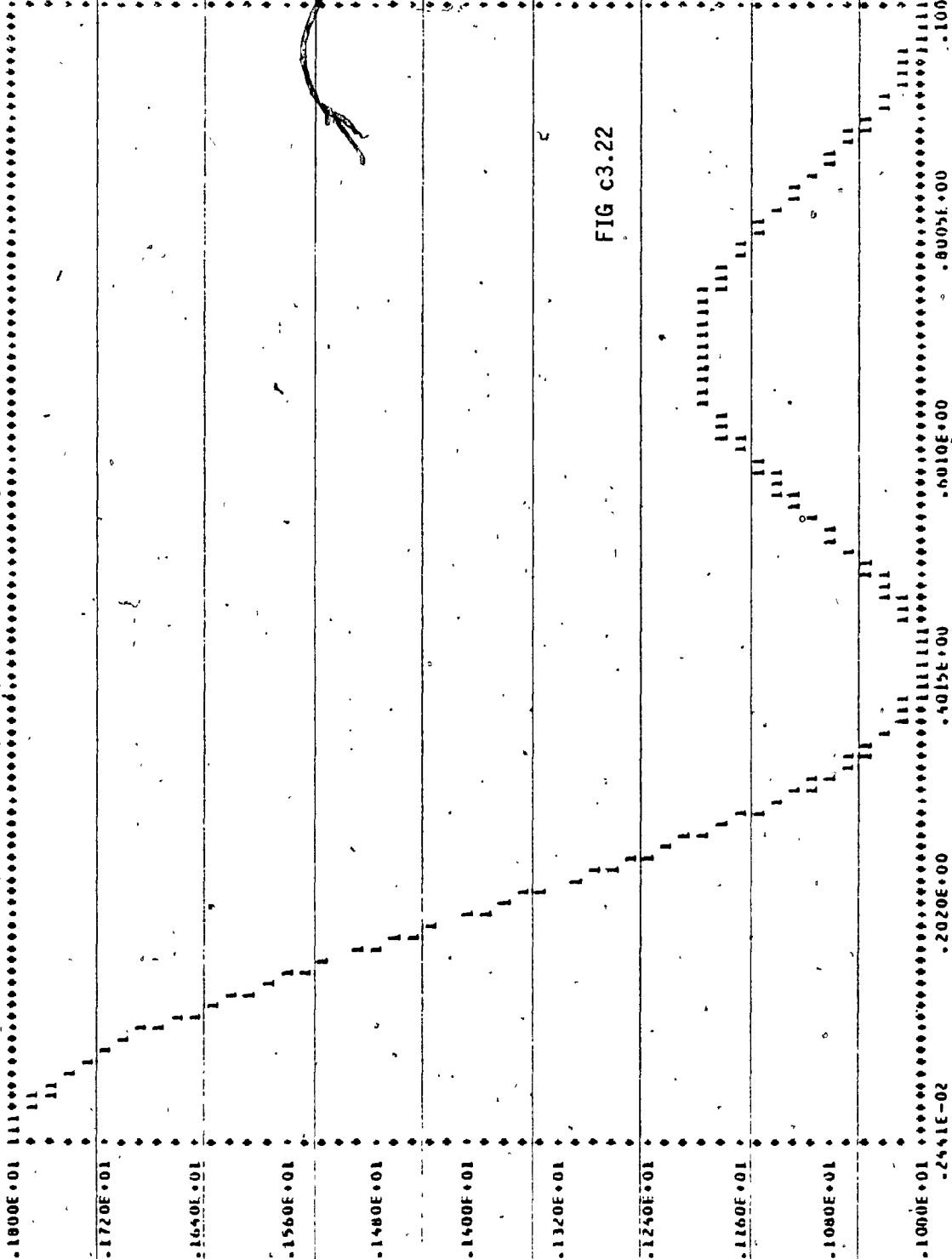
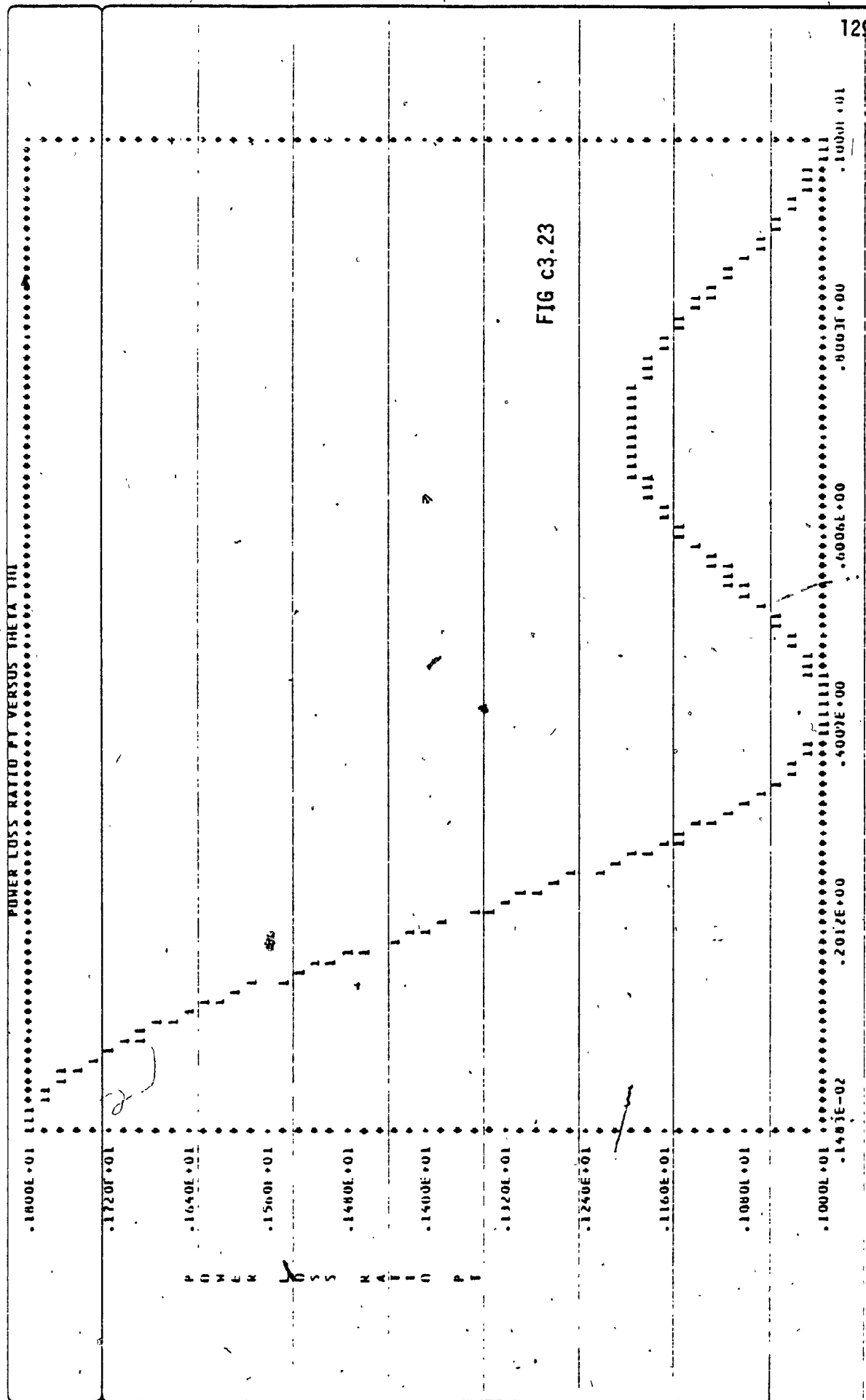


FIG C3.22

M= .40000000E+09 C= .20000000E-10 B= .30000000E-02	
Y1= .11256869E+01 R1= .12578430E+00	
NEW Z1= .17363246E+01 NEW Z2= .22360680E+01 NEW Z3= .23795299E+01	
EXTRA PHASE SHIFT PHI3= .10459158E+00 DISTANCE MOVED X3= .52295789E-01	
THE	PT
.33789266E-01	.17863498E+01
.27328083E-01	.17910518E+01
.20866900E-01	.17947739E+01
.14405717E-01	.17975062E+01
.79445344E-02	.17992409E+01
.14833515E-02	.17999735E+01
.49778314E-02	.17997019E+01
.11439014E-01	.17984269E+01
.17900197E-01	.17961519E+01
.24361380E-01	.17928832E+01
.30822563E-01	.17886296E+01
.37283746E-01	.17834030E+01
.43744929E-01	.17772175E+01
.50206112E-01	.17700900E+01
.56667295E-01	.17620400E+01
.63128478E-01	.17530893E+01
.69589661E-01	.17432625E+01
.76050843E-01	.17325860E+01
.82512025E-01	.17210889E+01
.88973209E-01	.17088023E+01
.95434392E-01	.16957593E+01
.10189558E+00	.16819950E+01
.10835675E+00	.16675464E+01
.11481794E+00	.16524521E+01
.12127912E+00	.16367528E+01
.12774031E+00	.16204894E+01
.13420149E+00	.16037057E+01
.14066267E+00	.15864458E+01



A = .50000000E+09 C = .20000000E+10 B = .10000000E+01

T1 = .11257668E+01 R1 = .12591883E+00

NEW Z1 = .17359199E+01 NEW Z2 = .22360680E+01 NEW Z3 = .28303172E+01

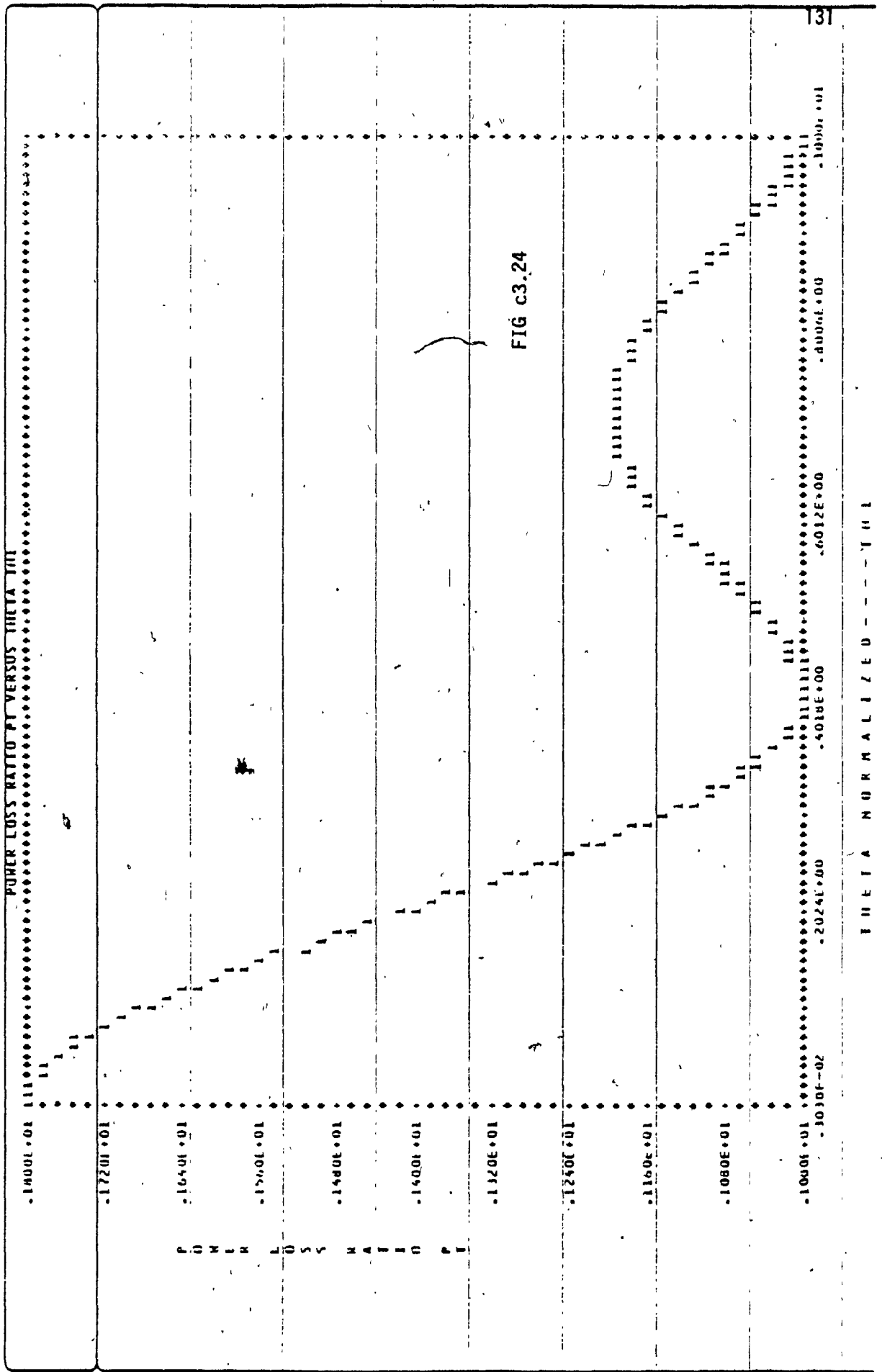
EXTRA PHASE SHIFT PH13 = .13070077E+00 DISTANCE MOVED X3 = .65150387E+01

THL

PT

.42593263E-01	.17787589E+01
.36067117E-01	.17847201E+01
.29590972E-01	.17997189E+01
.23034826E-01	.17937416E+01
.16513681E-01	.17967771E+01
.10002536E-01	.17988172E+01
.34863902E-02	.17998562E+01
.30297552E-02	.17998914E+01
.35459006E-02	.17989225E+01
.16062046E-01	.17969528E+01
.22578191E-01	.17939566E+01
.29094337E-01	.17900327E+01
.35610482E-01	.17851020E+01
.42126629E-01	.17792077E+01
.48642773E-01	.17723660E+01
.55158918E-01	.17645955E+01
.61675064E-01	.17559175E+01
.68191209E-01	.17463556E+01
.74707354E-01	.17359357E+01
.81223500E-01	.17246360E+01
.87739645E-01	.17126372E+01
.94255791E-01	.16998215E+01
.10077194E+00	.16862737E+01
.10724809E+00	.16720301E+01
.11380423E+00	.16571239E+01
.12032037E+00	.16416099E+01
.12683652E+00	.16255144E+01
.13333266E+00	.16088850E+01

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CHAPTER IV

SUMMARY AND DISCUSSIONS

In this report the Stepped Quarter Wave Transformer is considered and characteristic impedances are determined for three sections to give Butterworth and Chebyshev response in the passband. Both Stepped Quarter Wave Transformer with and without discontinuity capacitances are discussed. It is found that in both cases, analytical methods to determine the characteristic impedances is highly difficult so a computer aided analysis is to be employed. The characteristics are then plotted with the aid of a computer subroutine for both Chebyshev and Butterworth cases. The values of characteristic impedances which are determined are then used to plot the characteristic curves for both the cases and it is shown that the curves are exactly same as compared to the one plotted without using these values of characteristic impedances thus verifying that the values of characteristic impedances are determined correctly. The values of characteristic impedances are varied slightly and the effect is observed, it is seen that it does not make any changes in the behaviour of the curves.

Stepped Quarter Transformers with capacitance at junctions are discussed and their effects are observed. It is shown that there is a small increase in the magnitudes of individual step reflections and extra phase shifts are created in the reflection and transmission coefficients of the step due to this junction discontinuity. Methods

of correction are applied for these effects and a computer program is written for both Butterworth and Chebyshev cases which determines the values of Transmission and Reflection coefficients, susceptance and the new values of characteristic impedances along with the extra phase shift introduced and the distance moved in order to compensate these effects. These effects are tested in this program for different frequencies and capacitance values and it is shown that as the frequency increases the effect of discontinuity capacitance increases on the value of Transmission and Reflection coefficients, characteristic impedances and the extra phase shift also increases. Finally in this program response is plotted using computer subroutine showing the effects of discontinuity capacitance.

Therefore it can be concluded that this type of Stepped Quarter Wave Transformers can be designed with the aid of a computer, as analytical solution is very difficult.

It is suggested that similar studies can be conducted for half wave filters by the use of computers and the same work can be continued in the digital domain.

REFERENCES

- [1] Cohn, S. B. "Optimum Design of Stepped Transmission Line Transformers", *IRE Transactions, PGMTT-3*, pp. 16-21, April 1955.
- [2] Collin, R. E. "Theory and Design of Wide-Band Multisection Quarter-Wave Transformers", *Proceedings IRE 43*, pp. 179-185, February 1955.
- [3] Ghose R. N. *Microwave Circuit Theory and Analysis*. McGraw-Hill Book Company 1963.
- [4] Magnusson, P. C. *Transmission Lines and Wave Propagation*. Allyn and Bacon Series in Electrical Engineering 1965.
- [5] Matthae, G. L. *Microwave Filters, Impedance Matching Networks, and Coupling Structures*. McGraw-Hill Book Company 1964.
- [6] Slater, J. C. *Microwave Transmission*. McGraw-Hill Book Company 1942.